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Faculty of Applied Mathematics & Control Processes
THE INTERNATIONAL SOCIETY OF DYNAMIC GAMES
(Russian Chapter)

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ABSTRACTS

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The abstract volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

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ТЕОРИЯ ИГР И МЕНЕДЖМЕНТ. Сб. тезисов 6-ой международной конференции по теории игр и менеджменту / Под ред. Л.А. Петросяна и Н.А. Зенкевича. – СПб.: Высшая школа менеджмента СПбГУ, 2012. – 286 с.

Сборник содержит тезисы докладов участников 6-ой международной конференции по теории игр и менеджменту (27–29 июня 2012 года, Высшая школа менеджмента, Санкт-Петербургский государственный университет, Санкт-Петербург, Россия). Представленные тезисы относятся к теории игр и её приложениям в менеджменте.

Тезисы представляют интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

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Plenary Speakers

Sergey Aseev,	Institute of Mathematics RAS (Russia)
Michele Breton,	HEC Montreal (Canada)
Josef Hofbauer,	University of Vienna (Austria)
Ehud Kalai,	Northwestern University (USA)
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WELCOME ADDRESS

We are pleased to welcome you at the Sixth International Conference on Game Theory and Management (GTM2012) which is held in St. Petersburg University and organized by the Graduate School of Management (GSOM) in collaboration with the Faculty of Applied Mathematics & Control Processes and the International Society of Dynamic Games (Russian Chapter).

The Conference is designed to support further development of dialogue between fundamental game theory research and advanced studies in management. Such collaboration had already proved to be very fruitful, and has been manifested in the last two decades by Nobel Prizes in Economics awarded to John Nash, John Harsanyi, Reinhard Selten, Robert Aumann, Eric Maskin, Roger Myerson and few other leading scholars in game theory. In its applications to management topics game theory contributed in very significant way to enhancement of our understanding of the most complex issues in competitive strategy, industrial organization and operations management, to name a few areas.

Needless to say that Game Theory and Management is very natural area to be developed in the multidisciplinary environment of St. Petersburg University which is the oldest (est. 1724) Russian classical research University. This Conference was initiated in 2006 at SPbU as part of the strategic partnership of its GSOM and the Faculty of Applied Mathematics & Control Processes, both internationally recognized centers of research and teaching.

We would like to express our gratitude to the Conference's key speakers – distinguished scholars with path-breaking contributions to economic theory, game theory and management – for accepting our invitations. We would also like to thank all the participants who have generously provided their research papers for this event. We are pleased that this Conference has already become a tradition and wish all the success and solid worldwide recognition.

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Professor Sergei P. Kouchtch

Dean, Graduate School of Management

Professor Leon A. Petrosyan

Dean, Faculty of Applied

Mathematics & Control Processes

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WELCOME

On behalf of the Organizing and Program Committees of GTM2012, it gives us much pleasure to welcome you to the International Conference on Game Theory and Management in the Graduate School of Management and Faculty of Applied Mathematics & Control Processes of St. Petersburg University. This conference is the sixth of the St. Petersburg master-plan conferences on Game Theory and Management, the first one of which took place also in this city six years before. It is an innovated edition as to investigate the trend and provide a unique platform for synergy among business and financial systems, on one hand and industrial systems, on the other, in game-theoretic support of national economies in the recent process of globalization. Mathematical and especially game-theoretic modeling the globalized systemic structure of the world of the future, and managing its conduct towards common benefits is becoming a primary goal today.

This conference held in new millennium is not unique as the Sixth International Conference on Game Theory and Management since parallel to the conferences GTM2007, GTM2008, GTM2009, GTM2010 and GTM2011 other international workshops on Dynamic Games and Management were held worldwide. Because of the importance of the topic we hope that other international and national events dedicated to it will follow. Starting our activity in this direction six years before we had in mind that St. Petersburg University was the first university in the former Soviet Union where game theory was included in the program as obligatory course and the first place in Russia where Graduate School of Management and Faculty of Applied Mathematics were established.

The present volume contains abstracts accepted for the Sixth International Conference on Game Theory and Management, held in St. Petersburg, June 27-29, 2012. As editors of the Volume VI of Contributions to Game Theory and Management we invite the participants to present their full papers for the publication in this Volume. By arrangements with the editors of the international periodical Game Theory and Applications the conference may recommend the most interesting papers for publication in this journal.

St. Petersburg is especially appropriate as a venue for this meeting, being “window to Europe” and thus bridging the cultures of East and West, North and South.

Acknowledgements. The Program and Organizing Committees thanks all people without whose help this conference would not have been possible: the invited

speakers, the authors of papers, all of the members of Program Committee for referring papers, the staffs of Graduate School of Management and Faculty of Applied Mathematics & Control Processes.

We would like to thank Maria Dorokhina, Margarita Gladkova, Anna Tur, Tatyana Grigorova and Andrew Zyatchin for their effective efforts in preparing the conference.

We thank them all.

Leon A. Petrosyan, GTM2012 Program Committee

Nikolay A. Zenkevich, GTM2012 Organizing Committee

Stability and Index of the Meet Game on a Lattice

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Keywords: *Game Form, Social Choice, Stability, Stability index*

Stability is an essential requirement for political institutions; however it is both a theoretical and an empirical fact that political institutions are often unstable. Instability occurs when contradictory forces prevent the emergence of a persistent outcome, that is a state that is self-sustained and defensible when it is subject to attacks aiming to dismantle it. What can be observed when instability occurs is a volatile situation: any outcome that is proposed is subject to obstruction by some factions that block it. The result is a stalemate, where the institution is deadlocked. As far as the governance or the constitution of an established state is concerned, such a deadlock paves the way for a dramatic change of the institution itself; the latter is necessarily exogenous since no endogenous solution can be expected. In [2] and [3] the author argues that, however chaotic it may seem at first sight, instability in regulated systems presents some regularity and therefore there exist "patterns of instability".

Our basic model for analyzing interactions consists of a strategic game form G and some solution concept say E . We shall say that such a mechanism is stable if it has solutions for all preference profiles. The absence of solution results from conflicting relations between preferences of the agents and the opposition power that the game form allocates to active agents or coalitions. If groups of alternatives are merged together, the power distribution being preserved, then preferences become comparatively smoothed and a solution may emerge. Merging alternatives amounts to making a partition of the alternative set in such a way that agents are indifferent between elements of the same class. The minimal size of such a partition with no solution is the *stability index*. It is well known ([2] [3]) that instability for the pair (G, E) is closely related to the existence of some generalized Condorcet cycle, and that the stability index is equal to the minimal

size of such a cycle. In [2] some general properties related to the structure of instability have been shown for the core and the exact core. In [3] the study has been extended to Nash-like solutions including Nash proper and strong Nash.

In this paper we characterize stability and we compute the stability index of the *meet game form*.

Game forms: Solutions, Stability and Index

Let $G = \langle X_1, \dots, X_n, A, g \rangle$ be a strategic game form. The set of players is $N = \{1, \dots, n\}$, X_i is the strategy set of players i , $g : \prod_{i \in N} X_i \rightarrow A$ is the outcome function, assumed to be surjective. For any $S \in P_0(N)$ the product $\prod_{i \in S} X_i$ will be denoted X_S . $Q(A)$ will denote the set of all preorders on A , that is all binary relations on A which are transitive and complete. If $R \in Q(A)$ we denote by R° (resp. R^\cdot) the strict binary relation (resp. the equivalence relation) induced by R on A . Given any preference profile $R_N \in Q(A)^N$, the game form G induces a game $(X_1, \dots, X_n; Q_1, \dots, Q_n)$ with the same strategy spaces and where Q_i is the preorder on X_N defined by: $x_N Q_i y_N$ if and only if $g(x_N) R_i g(y_N)$ for $x_N, y_N \in X_N$. We denote this game by $G(R_N)$.

We shall adopt solution concepts in which a specified set of coalitions M may be active. Any $M \subset P_0(N)$ is called an *active coalition structure*. An alternative a is an *M -equilibrium outcome* of G at R_N if there exists some equilibrium $x_N \in X_N$ of $G(R_N)$ such that $g(x_N) = a$. We denote by $EO(M)(G, R_N)$ the set of all M -equilibrium outcomes of (G, R_N) . An alternative a is in the *M -exact core* of (G, R_N) if there is no coalition $S \in M$ with the following property : for any $z_N \in X_N$ such that $g(z_N) = a$ there exists $y_S \in X_S$ such that $g(y_S, z_{S^c}) R_i^\circ g(z_N)$ for all $i \in S$. Denote by $C_{1,M}(G, R_N)$ the M -exact core of (G, R_N) . An alternative a is in the *M - β -core* of (G, R_N) if there is no coalition $S \in M$ with the following property: for any $z_N \in X_N$, there exists $y_S \in X_S$ such that $g(y_S, z_{S^c}) R_i^\circ a$ for all $i \in S$. Denote by $C_{0,M}(G, R_N)$, the M - β -core of (G, R_N) .

Let Π be the set of all partitions of A . If $\pi \in \Pi$ and $a \in A$ we denote by $\pi(a)$ the class of the partition that contains a . Let $Q_*(\pi)$ be the set of all $R \in Q(A)$ such that whenever $\pi(a) = \pi(b)$ then aRb . Let $\pi, \pi' \in \Pi$. π *refines* π' , if every element of π is included in some element of π' . If π refines π' then $Q_*(\pi) \subset Q_*(\pi')$.

Let Π_r be the set of all partitions of A with r elements. Since any element of Π_r has a refinement in Π_{r+1} , then $\{Q_*(\pi) : \pi \in \Pi_r\} \subset \{Q_*(\pi) : \pi \in \Pi_{r+1}\}$ for any $r \geq 1$. Moreover $\{Q_*(\pi) : \pi \in \Pi_1\}$ is a singleton and $\{Q_*(\pi) : \pi \in \Pi_p\} = Q(A)$. We say that G is r - M -solvable if $EO(M)(G, R_N) \neq \emptyset$ for all $R_N \in Q_*(\pi)^N$ and all $\pi \in \Pi_r$. G is r - M -exactly stable if $C_{1,M}(G, R_N) \neq \emptyset$ for all $R_N \in Q_*(\pi)^N$ and all $\pi \in \Pi_r$. G is r - M - β -stable if $C_{0,M}(G, R_N) \neq \emptyset$ for all $R_N \in Q_*(\pi)^N$ and all $\pi \in \Pi_r$. We say that G is M -solvable if G is r - M -solvable for all $r \geq 1$. Similar definitions can be made for the M -exact core and the M - β -core.

Definition 1 The *stability index* of G relatively to the M -equilibrium (resp. M -exact core, resp. M - β -core) is the smallest integer $r \geq 1$ such G is not r - M -solvable (resp. r - M -exactly stable, r - M - β -stable) (with the convention that the index is $+\infty$ if no such integer exists).

The object of this paper is to give necessary and sufficient conditions for stability and to determine the stability index of the following game form, called the *meet game form* $\Gamma = \langle X_1, \dots, X_n, A, \mu \rangle$, where $X_1 = \dots = X_n = A$, A is a meet-semilattice (precise definitions are given below), and μ is the meet function that is:

$$\mu(x_1, \dots, x_n) = x_1 \wedge \dots \wedge x_n \quad (x_1 \in A, \dots, x_n \in A). \quad (1)$$

The meet game form

A *partially ordered set*, or *poset*, is a pair (A, \geq) where \geq is a binary relation on A that is reflexive, transitive and antisymmetric. A poset is a *meet semilattice* if any pair $\{x, y\} \subset A$ has an infimum, that is a greatest lower bound, denoted $x \wedge y$. The infimum of any family (x_1, \dots, x_k) will be denoted $x_1 \wedge \dots \wedge x_k$. In this section (A, \geq) is assumed to be a finite meet semilattice and $\Gamma = \langle X_1, \dots, X_n, A, \mu \rangle$ is the meet game form (1) defined on A . Γ has the following remarkable property:

Proposition 2 For any $R_N \in Q(A)^N$, an outcome is an M -equilibrium outcome of Γ if and only if it is in the M -exact core of Γ that is :

$$EO(M)(\Gamma, R_N) = C_{1,M}(\Gamma, R_N)$$

Corollary 3 The meet game form Γ is M -solvable if and only if it is M -exactly stable. The stability index of Γ is the same whether we consider the M -exact core or the M -equilibrium.

Thus studying stability of the local effectivity function is sufficient not only for M -exact stability of Γ , but also for its M -solvability. Here is its precise description for any M :

Definition 4 A nonempty subset $T \subset M$ has the *empty intersection property* if $\bigcap_{S \in T} S = \emptyset$. The *Nakamura Number* of M , denoted v_M , is the minimum of the cardinality of T where T describes all the subsets of M with the empty intersection property (with the convention that this number is $+\infty$ if no subset of M has the empty intersection property).

Steps and gaps of a binary relation

Let $N^* = \{1, 2, \dots\}$ denote the set of strictly positive natural numbers. For $q \in N^*$ let I_q denote the interval $\{1, \dots, q\}$, and Z/qZ the quotient of Z by the its additive subgroup qZ . Addition in Z/qZ is the addition modulo q . Let (A, \rceil) be a binary relation on A . A couple $(a, b) \in A \times A$ such that $a \rceil b$ will be called a *step*. Let $q \in N^*$. A q -enumeration of A is an injective mapping $e: Z/qZ \rightarrow A$. Let e be a q -enumeration of A . An e -edge is any ordered pair of the form $v = (e_k, e_{k+1})$ where $k \in Z/qZ$. Thus a 1-enumeration e has only one edge (e_1, e_1) . Two e -edges v and w are said to be *adjacent* if $v = (e_k, e_{k+1})$ and $w = (e_\ell, e_{\ell+1})$ and $k+1 = \ell$. An e -chain is any sequence $c = (v_1, \dots, v_r)$ of distinct e -edges such that v_k and v_{k+1} are adjacent ($k = 1, \dots, r-1$). The *length* of c is the number of its e -edges. It is denoted $|c|$. Since there is no repetition of edges in a chain: $|c| \leq q$. There are exactly q e -chains with length q , where only the initial vertex differ; we shall identify them all with e . An e -edge is an e -step if it is a step. We usually use the same notation for an e -chain (a sequence of e -edges) and the set of its edges. Thus $c \cap c' = \emptyset$ means that c and c' do not have common edges. Let c and c' be two e -chains such that $c' \subset c$. We say that

c' is a c -gap if c' contains no steps and if it is maximal for inclusion in c for this property. If c is an e -chain, we denote by $d(c)$ the number of e -steps in c , and $g(c)$ the number of c -gaps. It is easy to see that $d(c) + g(c) \leq |c|$. For $k \geq 1$, let C_e^k be the set of all e -chains such that $d(c) = k$. We introduce the following numbers related to the binary relation (A, \sqsubseteq) :

$\delta_A = \max_e d(e)$ where e describes all the set of p -enumerations.

$\gamma_e(k) = \min_{c \in C_e^k} g(c)$ with the convention $\gamma_e(k) = +\infty$ if $C_e^k = K\emptyset$.

$\gamma_A(k) = \min_e \gamma_e(k)$ where e describes all the set of p -enumerations.

δ_A will be called the *depth* of A , $\gamma_e(\cdot)$ will be called the *gap function* of e and $\gamma_A(\cdot)$ will be call the *gap function* of A . Remark that γ_e and γ_A are increasing functions. By convention $\gamma_e(+\infty) = \gamma_A(+\infty) = +\infty$.

A binary relation (A, \sqsubseteq) is said to be *acyclic* if for any $q \in \mathbb{N}^*$, any q -enumeration e contains at least one e -gap. If (A, \sqsubseteq) is acyclic, then in particular it is *irreflexive* that is one never has $x \sqsubseteq x$ for $x \in A$. To any poset (A, \geq) one can associate the binary relation $(A, >)$ where $x > y$ if and only if $x \geq y$ and $x \neq y$. $(A, >)$ is then an acyclic binary relation.

Theorem 5 For any M , the meet game form Γ is M - β -stable. Γ is M -exactly stable (and therefore M -solvable) if and only if either $N \notin M$ or $\delta_A < v_M$.

Stability and Index of the meet game form

In this section we assume that (A, \sqsubseteq) is an acyclic binary relation. We shall write $(a \sqsubseteq b)$ if $(a \sqsubset b)$ or $(a = b)$. For any $\emptyset \neq M \subset P_0(N)$, we consider the local effectivity function E_M defined as follows: For $U \in P_0(A)$:

$$E_M[U](S) = \begin{cases} \{B \in P_0(A) \mid \forall a \in U, \exists b \in B : a \sqsubseteq b\} & \text{if } S \in M, S \neq N \\ P_0(A) & \text{if } S \in M, S = N \\ \emptyset & \text{if } S \notin M \end{cases}$$

The corresponding effectivity function is defined by $E_{0,M}[U](S) = E_{0,M}(S) = E_M[A](S)$ ($S \in P(N)$).

Theorem 6 $E_{0,M}$ is stable for any M .

Theorem 7 E_M is stable if and only if either $N \notin M$ or $\delta_A < v_M$.

Theorem 8 Assume $N \in M$. We have the equality: $\sigma(E_M) = v_M + \gamma_A(v_M) + 1$.

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The Problem of the Realizability of the Function with Properties of the Golovach Function

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Keywords: *Guaranteed search, ε -capture, Search numbers, The Golovach function, Realizability*

Richard L. Breisch published an article ([3]) about organizing a search for a person moving in a cave. Consider a man that is lost in a dark cave. A team of rescuers is trying to find the lost man using their knowledge on the structure of the cave passages (which is modeled using a graph). The question is what is the minimum number of the searchers required to explore a cave so that it is impossible to miss finding the victim if he is in the cave. This problem was formalized by Parsons ([8]) (later but independently the equivalent problem was stated by Petrov ([9])) as a problem on a search number of a graph. And because of great number of applications, close connections with some graph invariants and other problems on graphs the Parsons-Petrov problem was extensively explored ([6]; [7]). The study on speleotopology (the word that was coined by Breisch for the applications of graph theory to study of caves) is also greatly expanded ([4]).

We consider one generalization of the Parsons-Petrov problem ([5]). Let G be a finite connected graph embedded in the 3-dimensional Euclidean space so that its vertices are represented by distinct points, and its edges are represented by polygonal curves with finite number of the line segments, and with endpoints at the vertices of the graph G . Unlike the cave exploration problem the lost person is considered as an invisible evader who, in turn, tries to escape. It is supposed that the evader is informed about a search plan of a team of pursuers before the beginning of the search. Thus the worst-case is considered, so the pursuers have to guarantee the capture of the evader without any probabilistic assumption on the game behavior. The trajectories of all agents are continuous (for G as a topological space with the induced topology) functions on any closed time intervals into G . We say that the evader is caught if the distance

between the pursuer and the evader is less than or equal to a given nonnegative number ε . The problem is to find the minimum number of pursuers that guarantee the capture of the evader. This problem is called ε -search problem, the required minimum number of the pursuers is called ε -search number, and the value of ε represents the radius of capture.

The problem mentioned in the title is stated as follows. A function f is specified on each nonnegative real number, is piecewise constant, non-increasing, right continuous, is imaged in positive integers, and starting with a certain sufficiently great argument is equal to 1. (The properties listed above are necessary to the Golovach function which assigns the ε -search number to each ε .) The problem of the realizability of the function f within a certain class of the topological graphs G is to find a graph $G \in \mathcal{G}$ with Golovach function equaled to f .

The most recent results on ε -search problem focus on a sequence of counterexamples disproving a hypothesis that on any planar graph by adding an extra pursuer to a current team we guarantee the capture with smaller radius of capture ([1]; [2]). In terms of ε -search problem it means that the Golovach function for any planar graph has only unit jumps. The problem of the realizability was stated so to simplify the proof of one hypothesis that Golovach function for a certain significant class of the graphs has only unit jumps. The hypothesis was disproved, but the problem of the realizability appear to be interesting to study. Some trivial cases could be fully examined. In more complicated cases the criteria of realizability of the function with properties of the Golovach function within special classes of the graphs could be proved. But in some rather simple cases the problem of the realizability becomes nontrivial.

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Advertising Match Signals Quality

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The paper studies the role of advertising in markets with products differentiated both vertically (different quality) and horizontally (different personal match) where the producer is not able to credibly communicate the quality of the good. We create and analyze a model where the producer chooses whether to advertise or hide personal match and this decision depends on the second dimension, which is internal quality of the good. We show that under certain assumptions there exists a separating equilibrium where the company that produces high quality product chooses to advertise “match” (horizontal characteristics), while the company that produces the standard quality product chooses to hide the horizontal characteristics.

By advertising the horizontal characteristics the company creates additional differentiation in the valuation of the good by consumers. We assume that by advertising “match” the company does not create additional value in aggregation, however, creates extra value for some customers. A certain group of consumers understand that this product is designed specifically for them, while others realize that there are characteristics they do not like. As a result, the valuation for some consumers increases while the valuation of others decreases. In other words, the “match advertising” leads to the rotation of the demand curve.

The choice whether to advertise the match or not is endogenous and based on the quality of the good the company produces and on the production cost.

We first show that the firm with high quality and high production cost benefits more from match advertising because it needs to concentrate on a narrow market with a very high price. Advertising will further increase quality for a narrow group of customers, and will allow the manufacturer to charge higher price. At the same time, the firm with low quality will concentrate on a wide market, because market contraction will hurt this firm.

Second, we show to separate itself from the low quality firm the high quality firm might need to increase the price higher than the perfect information price. If the price is low enough the possibility to mimic the behavior of the high-quality firm may be attractive for the low quality firm. The customers will believe that this product is of high quality and will pay more. To separate themselves from the low quality firm the high quality firm needs to increase the price, forgoing some profit due to loss of customers. However, mimicking the company that concentrates on a very narrow market will be unprofitable for the low quality firm, and, therefore, the separation is achieved.

Third, we find the parameter region where the separation is achieved without any distortions. Contrary to other results (e.g. Bagwell and Riordan (1991)) we show that high quality firm sometimes does not have to set its price higher than the perfect information price in order to signal the high quality of the product.

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A Parametric Family of three Ranked Objects Auctions: Equilibria and Associated Risk.

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Keywords: *Multi-object auctions, Bayesian Nash equilibrium, expected revenue, value at risk.*

This paper deals with simultaneous auctions of three commonly ranked objects following the model studied in Menezes and Monteiro (1998). For these problems we introduce a parametric family of auction mechanisms which includes the three classical auctions (discriminatory-price auction, uniform-price auction and Vickrey auction) and we call it the DUV family. We provide the unique Bayesian Nash equilibrium for each auction in DUV and prove a revenue equivalence theorem for the parametric family. Likewise, we study the value at risk of the auctioneer as a reasonable decision criterion to determine which auctions in DUV could be better for the auctioneer interests. We show that there are auction mechanisms in DUV better than the classic auction mechanisms with respect to this criterion. In this paper we are interested in multi-object auction, in particular, when the objects are commonly ranked. This kind of auctions are used in numerous contexts such as licenses allocation (see, for example, Janssen et al. 2010), keywords auctions in search engines on Internet (see, for example, Feng et al. 2007; Lim and Tang, 2006; Aparicio et al., 2010), electricity markets (see, for example, von der Fehr and Harbord, 1993; Alonso and Tejada, 2010), the sale of raw materials or other products (minerals, fish, flowers, etc.) , ... We are going to consider simultaneous auctions of three ranked objects where each bidder is only interested in just one of those ranked object and submit exactly one single bid.

Simultaneous auctions of ranked objects have been studied in the literature in several papers analyzing different aspects of them. Menezes and Monteiro (1998) studied simultaneous pooled auctions and analyzed, in particular, the discriminatory-price auction for k ranked objects and n ($n > k$) bidders who only submit a single bid that they will pay whenever one of the k object was gained, i.e., if their bids were among the k highest bids. On the other hand, Janssen et al. (2010) modelled the Irish allocation rule for two ranked objects and n bidders and the bidders submit a complex bid with two bids, one for each object. In contrast with Menezes and Monteiro (1998), the authors showed that simultaneous pooled auction with multiple bids do not have, in general, efficient equilibria, i.e., the objects do not always go to those who value them the most.

However, as Feng et al. (2007) claim, one important problem that is not analyzed in Menezes and Monteiro (1998) is the ex post loss problem, the so-called winner's curse (see, for example, Bazerman and Samuelson, 1983). In order to avoid the problem of not having efficient equilibria, Feng et al. (2007) followed the model introduced by Menezes and Monteiro (1998) but they modified it by enforcing a cutoff type (reserve price), obtaining a new auction mechanism called Tailored VCG Mechanims with the same expected revenue but without the ex post loss problem. In this paper, we also consider a simultaneous pooled auction similarly as in Menezes and Monteiro (1998) and hence the bidders only submit a single bid, but we solve the problem of ex post losses by allowing a bidder to pay only a part of his bid when does not win the highest-ranked object, instead of enforcing a reserve price as in Feng et al. (2007). In this way, the prices to be paid by the winning bidders are calculated from the list of bids by using a couple of parameters which are used to measure the influence of the highest bids in the determination of the final prices. These parameters play a role for the auctioneer in the decision problem of selecting the "best" auction for her interests.

Three classical and well-known auction mechanisms are the uniform price, the discriminatory price and the Vickrey auction (Vickrey, 1961). Using the above mentioned parameters we construct a parametric family of simultaneous auction of three ranked objects and n bidders which contains the three previous classical auctions. We prove that all members of this family have efficient equilibria and do not exhibit an ex post loss problem. In particular, we provide, in a compact result, the unique (symmetric) Bayesian Nash equilibrium for each auction belonging to this family. On the other hand, the reason why these auctions do not have the ex post loss problem is because of a bidder may pay only a portion of their original bid depending on the object gained.

Furthermore, this parametric family is maximal, in the following sense, if we consider other parameters then the corresponding auction either does not have efficient equilibria or has ex post loss problems.

On the other hand, when a single object is auctioned, the Revenue Equivalence Theorem (Myerson, 1981) establishes that, under some conditions, the seller obtains the same expected revenue from any auction in which the object always goes to the bidder with the highest valuation and every bidder would expect zero utility if his valuation were 0. Likewise, under some conditions, when multiple or heterogeneous objects are auctioned, the three classical auction mechanisms mentioned before, generate the same expected payment for the bidders and, therefore the same expected revenue for the auctioneer (see, for example, Krishna, 2002). In this paper, we extend this result by proving that every auction mechanism belonging to the parametric family verifies the corresponding Revenue Equivalence Theorem and, therefore all auctions in the parametric family generate the same expected revenue for the auctioneer.

In the literature we can find a number of papers about the optimal choice or design of auctions, usually from the point of view of the expected revenue (see, for example, Myerson, 1981; Riley and Samuelson, 1981; Feng, 2008). For that, if an auctioneer only takes into account the expected revenue (risk-neutral auctioneer), then every auction mechanism satisfying the Revenue Equivalence Theorem is indifferent for her. Nevertheless, an auction mechanism should be selected. Consequently, the auctioneer should have into account other criteria for making her decision. A possible criterion is collusion (Robinson, 1985), another is the variability (Vickrey, 1961), and other criteria could be used. Regarding the variability criterion, Vickrey (1961) calculated the variance for first-price and English auctions. Waehrer et al. (1998) proved that a risk-averse auctioneer prefers first-price auction to second-price auction, and second-price auction to English auction. Beltrán and Santamaría (2006) used simulation to analyze the variation for several auction mechanisms with the same expected revenue. And Krishna (2002) proved that the price distribution in second-price auction is a mean-preserving spread of the price distribution in first-price auction.

Although variance is a reasonable measure for the variability of a random variable, perhaps it is not suitable for comparing auction mechanisms, because variance does not indicate the sign of the revenue deviations when revenue can be volatile and reaches sudden extreme values both larger and smaller than the expected revenue. Hence, since an auctioneer is not affected if revenue is higher than expected, it seems

reasonable to be only interested in obtaining a measure for the risk of losses for each auction mechanism with respect to its expected revenue. In this sense, the value at risk (VaR), for example, measures the worst loss at a given confidence level and reflects how much can be lost with respect to the expected revenue at a certain probability (see, for details, Holton, 2004). Therefore the VaR for auctioneer revenue at a given confidence level quantifies the maximum loss with respect to the expected revenue. Accordingly to our previous comments, we believe that VaR could be very useful for helping auctioneer to identify which auctions mechanisms could be better for his interests when they are identical in terms of expected revenue but different in terms of risk losses. Thus, in order to establish a comparison among the auction mechanisms belonging to our parametric family, we calculate their VaR for auctioneer revenue and show that, under some conditions, there are new auction mechanisms which have lower VaR than any other classical auction mechanism.



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Entry in Multi-Activity Contests

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Keywords: *Contest success function, Complementarity, Rent dissipation, Entry, Caps*

In this paper, we extend our previous analysis of multi-activity contests (Maria Arbatskaya and Hugo M. Mialon, "Multi-Activity Contests" 2010, Economic Theory 43, 23-43 and "Dynamic Multi-Activity Contests" forthcoming, Scandinavian Journal of Economics) to endogenize entry. In the model, players simultaneously choose whether to enter a contest, and entrants then simultaneously choose efforts in multiple activities to win the contest. Players are asymmetric in their costs of effort in the activities. We find that greater complementarity between the efforts in the different activities increases rent-dissipation and reduces entry in the contest. We also find that capping one of the activities is more likely to reduce rent dissipation and increase entry in the contest if players are sufficiently symmetric in their effort costs in the activity.

We explore two applications of the model, one to corruption in government contracting and one to political campaign contributions. In competition for government contracts, firms may increase their chances of winning a contract by improving their efficiency or competitiveness, but they may also do so by bribing government officials. Since bribery and competitiveness efforts are likely to be complementary (i.e., bribery is more likely to succeed for a more competitive firm), our model suggests that laxer laws against bribery may reduce entry and competition for government contracts.

In political campaigns, supporters or political action committees can contribute to their preferred candidates directly or independently. In the U.S., caps have been imposed on both direct and independent contributions to political campaigns. However, the U.S. Supreme Court has recently ruled (in the case of Citizens United vs. FEC, 558 U.S. 08-205, 2010) that caps on independent contributions are unconstitutional. Given that asymmetry across supporters in their opportunity costs of independent contributions

is typically large (with rich supporters having much lower opportunity costs), our model suggests that uncapping independent contributions to political campaigns may reduce the number of candidates running for office.



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GAMES AND ECONOMIC BEHAVIOR

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Lorenz and Lexicographic Maximal Allocations for Bankruptcy Problems¹

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Keywords: *Bankruptcy problems, Lorenz criterion and lexicographic criterion*

We investigate the use of egalitarian criteria to select allocations in bankruptcy problems. In our work, we characterize the sets of Lorenz maximal elements for these problems. We show that the allocation selected by the Proportional Rule is the only allocation that belongs to all these Lorenz maximal sets. We prove that the Talmud Rule selects the lexicographic maximal element within a certain set. We introduce and analyze a new rule for bankruptcy problems that shares strong similarities with the Talmud Rule.

Bankruptcy problems

The tuple (N, d, E) is a bankruptcy problem if:

- N is a finite nonempty set.

- $\sum_{i \in N} d_i > E$.

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N represents the set of *claimants*, $E \in \mathbb{R}_+$ represents the amount to be divided, and $d \in \mathbb{R}_+^N$ is a vector of claims whose i -th component is d_i . An allocation to the claimants is represented by a real valued vector $x \in \mathbb{R}^N$ that satisfies $\sum_{i \in N} x_i = E$. An allocation x satisfies claim boundedness and non negativity if $d_i \geq x_i \geq 0$ for all $i \in N$.

We denote by $F(N, d, E)$ the set of allocations that satisfy claim boundedness and non negativity.

A sharing rule ϕ is a mapping that associates a vector

$\phi(N, d, E) \in F(N, d, E)$ with every problem (N, d, E) . Some well-known sharing rules are:

Constrained Equal Awards (CEA). This solution divides the endowment equally among the agents under the constraint that no claimant receives more than his \ her claim.

Constrained Equal Losses (CEL). This solution divides the total loss $(\sum_{i \in N} d_i - E)$ equally among the agents under the constraint that no claimant receives a negative amount.

The Proportional Rule (PR). This solution divides the endowment among the claimants proportionally to their claims.

Some well-known properties of a rule ϕ are the following.

- ϕ satisfies **order preservation for awards and losses** if for each (N, d, E) in Γ we have that $\phi(N, d, E)$ is order preserving for awards and losses. An allocation x is order preserving for awards and losses if $d_i \leq d_j$ implies that $x_i \leq x_j$ and $d_i - x_i \leq d_j - x_j$.

- ϕ satisfies **consistency** if for any problem (N, d, E) and any $S \subset N$ it holds that $\phi_i(S, (d_i)_{i \in S}, \sum_{i \in S} \phi_i(N, d, E)) = \phi_i(N, d, E)$ for all $i \in S$.

- ϕ satisfies **λ -claim boundedness (λ -CB)** if for any $(N, d, E) \in \Gamma$ we have that either $\phi_i(N, d, E) \geq \lambda d_i$ for all $i \in N$ or $\phi_i(N, d, E) \leq \lambda d_i$ for all $i \in N$.

Egalitarian Criteria

For any vector $z \in \mathbb{R}^d$ we denote by $\theta(z)$ the vector that results from z by permuting the coordinates in such a way that $\theta_1(z) \leq \theta_2(z) \leq \dots \leq \theta_d(z)$. Let $x, y \in \mathbb{R}^d$.

We say that the vector x *Lorenz dominates* the vector y (denoted by $x \succ_L y$) if $\sum_{i=1}^k \theta_i(x) \geq \sum_{i=1}^k \theta_i(y)$ for all $k \in \{1, 2, \dots, d\}$ and if at least one of these inequalities is strict. The vector x *weakly Lorenz dominates* the vector y (denoted by $x \pm_L y$) if $\sum_{i=1}^k \theta_i(x) \geq \sum_{i=1}^k \theta_i(y)$ for all $k \in \{1, 2, \dots, d\}$.

We say that the vector x *lexicographically dominates* the vector y (denoted by $x \succ_{lex} y$) if there exists k such that $\theta_i(x) = \theta_i(y)$ for all $i \in \{1, 2, \dots, k-1\}$ and $\theta_k(x) > \theta_k(y)$.

We say that the vector x *lexmax dominates* the vector y (denoted by $x \succ_{lm} y$) if there exists k such that $\theta_i(x) = \theta_i(y)$ for all $i \in \{k+1, k+2, \dots, n\}$ and $\theta_k(x) < \theta_k(y)$.

The set of awards-losses vectors

Let (N, d, E) be a problem and let x be an allocation. Each agent measures x_i in two ways. In one sense x_i measures how much he/she receives. In the other sense, $d_i - x_i$ measures how much he/she does not receive. Given the allocation x we define its associated ordered vector of awards-losses as follows:

$$x^{AL} = (x_1, \dots, x_n, x_1 - d_1, \dots, x_n - d_n).$$

We also use the following notation:

$$x^A = (x_1, \dots, x_n) \text{ and } x^L = (x_1 - d_1, \dots, x_n - d_n).$$

In this vector, awards and losses are equally weighted and equally treated. We also consider vectors where awards and losses are not equally treated. Given the allocation x we define its associated weighted vector of awards-losses as follows:

$$\lambda - x^{AL} = ((1-\lambda)x_1, \dots, (1-\lambda)x_n, \lambda(x_1 - d_1), \dots, \lambda(x_n - d_n))$$

where $\lambda \in [0,1]$. Note that $\lambda - x^{AL}$ with $\lambda = \frac{1}{2}$ is the vector of equal weights, which in our study is equivalent to considering x^{AL} or $\lambda - x^{AL}$ with $\lambda = \frac{1}{2}$. The following set

$$|\lambda - AL(N, d, E)| = \{|\lambda - x^{AL}| : x \in F(N, d, E)\}$$

is the set of vectors of awards-losses taken in absolute terms. Note that we use the notation $\lambda - x^{AL}$ instead of $\lambda - x^{AL}(N, d, E)$. We consider there is no confusion, so we prefer the notation $\lambda - x^{AL}$ for the sake of simplicity.

The Lorenz criterion

The set of Lorenz undominated allocations is defined as follows:

$$L(N, d, E) = \left\{ x \in F(N, d, E); \text{there is no } y \in F(N, d, E) \right. \\ \left. \text{such that } y^{AL} \succ_L x^{AL} \right\}.$$

Theorem 1 9 *The Lorenz maximal set coincides with the set of all allocations that satisfy order preservation in both ways: awards and losses.*

Corollary 2 10 *Let $x \in L(N, d, E)$. Then $(x_i)_{i \in S} \in L(S, (d_i)_{i \in S}, \sum_{i \in S} x_i T)$*

The weighted Lorenz maximal set for $\lambda \in [0,1]$ is:

$$\lambda - L(N, d, E) = \left\{ x \in F(N, d, E); \text{there is no } y \in F(N, d, E) \right. \\ \left. \text{such that } \lambda - y^{AL} \succ_L \lambda - x^{AL} \right\}.$$

For $\lambda \in (0,1)$ the weighted Lorenz maximal sets coincide.

The set $\lambda - L_{AT}(N, d, E)$ is defined as follows:

$$\lambda - L_{AT}(N, d, E) = \left\{ x \in F(N, d, E); \text{there is no } y \in F(N, d, E) \right. \\ \left. \text{such that } |\lambda - y^{AL}| \succ_L |\lambda - x^{AL}| \right\}.$$

Theorem 3 11 *The set $\lambda - L_{AT}(N, d, E)$ coincides with the set of all allocations that satisfy λ -claim boundedness and order preservation in both ways: awards and losses.*

Corollary 4 12 $\bigcap_{\lambda \in (0,1)} \lambda - L_{AT}(N, d, E) = \{PR(N, d, E)\}.$

The Lexicographic criterion

A central rule in the literature of bankruptcy problems is the Talmud Rule introduced by Aumann and Maschler (1985).

Let (N, d, E) be a bankruptcy problem. Then

$$T_i(N, d, E) = \begin{cases} \min\left\{\frac{d_i}{2}, \alpha\right\} & \text{if } E \leq \frac{\sum_{n \geq i} d_n}{2} \\ \frac{d_i}{2} + \max\left\{\frac{d_i}{2} - \alpha, 0\right\} & \text{otherwise} \end{cases}$$

where α is chosen such that $\sum_{n \geq i \geq 1} T_i(N, d, E) = E$.

Theorem 5 13 Let (N, d, E) be a bankruptcy problem. Then

$$T(N, d, E) = \left\{x \in F(N, d, E); |x^{AL}| \succ_{lex} |y^{AL}|, \text{ for all } y \in F(N, d, E)\right\}.$$

Aumann and Maschler (1985) characterize the Talmud Rule as the unique consistent rule for bankruptcy problems (Theorem A). In their work consistency is also called CG-consistency.

The *contested garment principle* is a solution used to solve two-claimant problems.

We introduce a new rule for bankruptcy problems, the Lexmax rule..

Definition 6 14 Let (N, d, E) be a bankruptcy problem. Then

$$LM(N, d, E) = \left\{x \in F(N, d, E); |x^{AL}| \succ_{lm} |y^{AL}|, \text{ for all } y \in F(N, d, E)\right\}.$$

This rule satisfies consistency, order preservation (in both ways) and $\frac{1}{2}$ -CB since it provides allocations that belong to the set $L_{AT}(N, d, E)$.

Let (N, d, E) be a bankruptcy problem. Then

$$LM_i(N, d, E) = \begin{cases} \max\left\{\min\left\{\frac{d_i}{2}, d_i - \alpha\right\}, 0\right\} & \text{if } E \leq \frac{\sum_{n \geq i} d_n}{2} \\ \min\left\{\max\left\{\alpha, \frac{d_i}{2}\right\}, d_i\right\} & \text{otherwise} \end{cases}$$

where α is chosen such that $\sum_{n \geq i \geq 1} T_i(N, d, E) = E$.

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Journals in Game Theory

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SPRINGER



The SD-prenucleolus for TU Games

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Keywords: *TU games, Prenucleolus, Per capita prenucleolus*

We introduce and characterize a new solution concept for TU games.

The new solution is called SD-prenucleolus and is a lexicographic value although is not a weighted prenucleolus. The SD-prenucleolus satisfies several desirable properties and is the only known solution that satisfies core stability, strong aggregate monotonicity and null player out property in the class of balanced games.

The SD-prenucleolus is the only known solution that satisfies core stability, continuity and is monotonic in the class of veto balanced games.

Semieulerian Cycles in Graphs and their Applications to Dynamical Searching Game

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One of the variants of searching problem in graphs was posed and began studied by T.D. Parsons [1] and N.N.Petrov [2] independently (survey of the results up to 2007 y. see [3]). If G is the connected geometrical graph with rectifiable edges then an absolutely continuous function $x : [0, +\infty) \rightarrow G$ (respectively $y : [0, +\infty) \rightarrow G$) satisfying the condition $|\dot{x}(t)| \leq \rho$ ($|\dot{y}(t)| \leq \sigma$) a.e. is called a trajectory of Searcher (of Hider; ρ and σ are given positive numbers). One can find a positive k_G such that if $\rho > k_G \sigma$ then Searcher will win the game of searching in the following sense: there exists a trajectory $x(\cdot)$ of Searcher that for any trajectory $y(\cdot)$ of Hider $x(t) = y(t)$ at some $t, t > 0$ and if $\rho < k_G \sigma$ Hider will win. It is known some estimations for k_G . For example in [4] three estimations were suggested. The most effective of them is based on so called semieulerian cycles, i.e. cycles passing throw all edges though not necessarily once exactly. Here we point out some index of eulerianity of a graph that allows to improve the estimation mentioned above.

Obviously there are semieulerian cycles in every graph. Let Σ be the set of such cycles. If $C \in \Sigma$ then its length will be written as $|C|$. Besides the number of all edges of a graph Γ will be denoted $|\Gamma|$. Then the quantity

$$e(\Gamma) = \min_{C \in \Sigma} \frac{|\Gamma|}{|C|}$$

will be called the index of semieulerianity of the graph Γ . Obviously $e(\Gamma) \leq 1$ and $e(\Gamma) = 1$ means the graph Γ is Eulerian. Further as every connected graph becomes Eulerian if one doubles its edges, therefore $\frac{1}{2} \leq e(\Gamma)$.

Theorem. $e(\Gamma) = \frac{1}{2}$ if and only if when Γ is a tree.

If $|C^*| = e(\Gamma)$ then C^* will be an optimal semieulerian cycle in Γ . Every optimal cycle generates some searching trajectory [2. 4]. This allows to establish the upper estimation for k_G by $e(\Gamma)$ and degrees of vertices of Γ .

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РОССИЙСКИЙ ЖУРНАЛ МЕНЕДЖМЕНТА

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Informed Seller in a Hotelling Market

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Keywords: *Informed Principal, Horizontal differentiation, Information disclosure*

We consider the problem of a monopolist seller who is selling a good to a potential buyer and is privately informed about some of the good's attributes. We focus on the case where goods with different attributes are horizontally differentiated: they appeal differently to different types of the buyer. Such a market is represented via a standard Hot model, where the seller's and the buyer's private information are locations on the Hotelling line and the buyer's utility from consuming the good is given by a base consumption value minus a cost depending on his distance from the seller.

We derive the optimal selling mechanism as the solution of an informed principal problem. We show that if the base value is sufficiently high, then it is optimal for the monopolist not to disclose any information, if the base value is sufficiently low, then it is optimal for her to reveal her private information. For intermediate base values, the monopolist gains by price discriminating the buyer's types over their value for information. A crucial feature of the optimal mechanism is that different information content is disclosed by the seller to different types of the buyer.

When the cost is linear or concave, the optimal mechanism can be implemented as a two-item menu: buy an opaque good without disclosure, or buy the information about the good with the option of purchasing the good afterwards at a predetermined exercise price. When the cost is convex, a continuum of offerings is optimal: buy the good with full disclosure; buy the opaque good without disclosure; or buy one of a continuum of options, each providing the right to return the good with probabilities that depend on the good's type.

Reflexive Behavior in Topology Control for Ad Hoc Networks

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Keywords: *Topology control, Ad hoc network, Price of anarchy, Reflexive behaviour*

We consider the problem of topology control in wireless ad hoc network. There is a set of mobile wireless devices able to adapt their transmitting ranges according to the network state. Nodes should dynamically adjust their transmitting ranges in order to maintain some global network property (e.g. connectivity) while reducing energy consumption or interference. Nodes behavior in noncooperative framework is examined. We study efficiency of applying reflexive strategies, taking into account the behavior of other nodes.

Introduction

Networks composed of mobile and independent wireless devices have been called ad hoc networks [1]. Nodes in ad hoc network functioning in decentralized manner often under uncertainty about global network state. During the topology control process nodes dynamically adjust the transmitting ranges of their radios in order to maintain such global properties as connectivity while minimizing some global objective function like energy consumption or interference level [2]. A detailed survey of topology control algorithms can be found in [2].

In distributed wireless networks coordination among distant nodes incurs additional overhead on network capacity and interference level. So algorithms where nodes don't need to coordinate their actions but optimizing only local objective functions are of great research and practical interest. These formulations lie in the sphere of non-cooperative game theory which has been widely used in such applications like Internet traffic prediction [3], base station placement in wireless telecommunications [4] and many others [5]. Game-theoretic formulation of topology control was studied in [6-8].

Algorithms of nodes behavior proposed in [6-8] are based on different modifications of myopic best response strategy. Such kind of behavior corresponds to agents with zero level of reflection in [9] because they don't take into account possible reaction of others. There is research interest in employing strategies with non-zero level of reflection. Effectiveness of agents with higher level of reflection in the frameworks of building emergency evacuation, market competition and group path planning in hostile environment were studied in [10]. In this paper we study the effectiveness of reflexive behavior in the topology control problem.

Topology control

Let us describe the *topology control* problem. There are a set of spatially distributed wireless devices. Each device can change the transmitting range of their radio. We consider the case of omnidirectional antennas hence all devices fall into the range can receive messages.

Ranges of all nodes constitute vector $\bar{r} \in R_+^n$ which induce an undirected *communication graph* $G = G(V, \bar{r})$ or $G = G(V, E)$. An edge $e = (u, v)$ is included in the communication graph if bidirectional communication between nodes u and v is possible. So both u and v must fall into the transmitting range of each other or $r_u \geq d(u, v)$ and $r_v \geq d(u, v)$, where $d(u, v)$ is Euclidian distance between u and v .

The goal of topology control is to maintain global connectivity of the communication graph and optimizing some global objective function. In [6-8] nodes aim to reduce their energy consumption. So the global network objective is to minimize the energy cost:

$$C(\bar{r}) = \sum_v r_v^\alpha \quad (1)$$

where r_v – radius of nodes v , $\alpha \geq 2$ is a distance-power gradient [1]. In the ideal case $\alpha = 2$ but in realistic situations α may vary in from 2 to 6.

Game-theoretic algorithms and reflexive behavior

Different non-cooperative algorithms based on nodes myopic best response strategy was proposed in [6-8]. This process occasionally leads to inefficient Nash equilibria [5] which may have cost of $O(n^\alpha)$ times more than optimal [6]. The ratio between the worst-case equilibrium and the global optimum is called price of anarchy [3]. The question is does it possible to avoid such inefficient equilibria via some reasoning about other nodes possible responses. Employing some hypothesis about other

participants' behavior is called non-zero level of reflection in [9]. The effectiveness of strategies with non-zero levels of reflection is investigated in this paper.

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Network Game of Pollution Cost Reduction

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Keywords: *Pareto Optimality, Time consistency, Payoff Distribution Procedure*

In this paper a n -person network game theoretical model of emission reduction is considered. Each player has its own evolution of the stock of accumulated pollution. Dynamic of player i , $i = 1, 2, \dots, n$ depends on emissions of players $k \in K_i$, where K_i is the set of players which are connected by arcs with player i . Nash Equilibrium is constructed. The cooperative game is considered. As optimal imputation the proportional solution is proposed.

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Gaming Model of Herd Control

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Keywords: *Collective behavior, Threshold model of decision making, Mob control*

Threshold model of a group of agents (the mob) is considered. These agents make binary decisions to act or not to act, taking in to account the decisions of the rest of the group. The problem of control over agents' thresholds is formulated and solved. The aim of this control is to reduce the number of agents that decided to act in the mob.



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WORLD SCIENTIFIC



Condorcet Consistency and Competition on Median Spaces

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Keywords: *Median voter theorem, Condorcet winner, Generalized single-peakedness, Median spaces, Hotelling*

The median voter theorem is one of the central theorems in economic theory. It applies to environments where agents have to choose one element out of a set of alternatives and states the following:

If preferences are single-peaked (on a line), then the median voter's favorite alternative wins in majority voting against any other alternative, i.e. it is a Condorcet winner.

This is not only a statement about the existence of a Condorcet winner, but also a characterization of it: the Condorcet winner coincides with the preference peak of the median voter. Since the domain of single-peaked preferences on a line is quite narrow, it is natural to ask to which extent this assumption can be relaxed. In a seminal contribution Nehring & Puppe [2007, The structure of strategy-proof social choice I: General characterization and possibility results on median spaces. JET 1351, 269-305] show how another result on single-peaked preferences on lines can be extended to a much larger class of preferences---single-peaked preferences on median spaces (allow for preference aggregation that is neither manipulable nor dictatorial). This interesting class of preferences covers a variety of intriguing examples since median spaces not only contain lines but also products of lines (e.g. grids), trees, and hypercubes as special cases. While existence of a Condorcet winner cannot be guaranteed for this large class of preferences, we show that the characterization given by the median voter theorem extends: if there exists a Condorcet winner, then it coincides with the median alternative (the "median voter").

Based on this result, we solve two non-cooperative games of political competition which are direct extensions of the classic Hotelling--Downs model to generalized single-peaked preferences on median spaces. An equivalent formalization in graph-theoretic terms is possible.

Forming Opinions in Social Networks: the Effects of Strategic Interaction

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Keywords: *Opinion dynamics, Social influence, Networks, Consensus, Conformity, Opinion leadership*

We study a model of opinion formation based on repeated averaging (also known as DeGroot model). In contrast to the literature, we assume that true opinions of other individuals cannot be observed and introduce the possibility that agents may state an opinion different from their true opinion, i.e. are dishonest. Assuming that the agents - endowed with heterogeneous, but quadratic preferences - play a Nash equilibrium in every period, leads to a substantive extension of the classic DeGroot model. The opinion dynamics can be concisely expressed by a derived matrix, which corresponds to a network of social influence. Inspecting the eigenvalues enables us to study the following questions: Does dishonesty foster or hinder convergence of opinions (e.g. to consensus in a society)? How do the long-run opinions in a scenario of dishonesty differ from a scenario of honesty? How does the group structure depend on the dishonest behavior?

Endogenous Productivity under Monopolistic Competition

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Keywords: *Investments, Monopolistic competition, Market integration, Relative love for variety, Country size*

We study a monopolistic competition model with general (unspecified) utility and cost functions, seeking for impact of market size on productivity or/and product quality. Both are endogenous, i.e., a producer can invest in decreasing marginal cost or in increasing quality. It turns out that such investments increase with the market size if and only if utility shows increasing «relative love for variety» which is the elasticity of the inverse demand.

The cross-countries differences in productivity are noticeable and allow for various explanations. In Industrial Organization, market structure and innovations were discussed a lot. Most close to our topic are empirical studies of relation between competitiveness and innovations. Many papers find here positive correlation. On the other hand, some papers demonstrate the possibility of non-monotone (inverted U -type) correlation between these values. Despite these evidences, typical theoretical oligopoly settings of industrial organization predict that innovation should decline with competition. Probably, this contradiction between theory and evidence stems from unrealistic assumption of fixed number of firms. To overcome it, Vives (2008) does model oligopolistic competition with free entry and show possibility of positively related competition and innovations.

To enrich theoretical explanations of these and other empirical regularities, we follow Dixit-Stiglitz-Krugman's monopolistic competition model, but step aside from its popular version with constant elasticity of substitution (CES) utility function, broadly criticized in the literature. Instead of exploring other specific utilities (quadratic,

exponential, etc.) - we find it more interesting to construct a general theory. Following Zhelobodko et. al. (2010) methodology, we link the patterns of market outcomes with certain properties of (unspecified) utility and cost functions.

More specific goal is to model endogenous choice of technology in a diversified sector with free entry. We develop the general-utility monopolistic competition models from Zhelobodko et. al (2010) and Zhelobodko et. al (2011) towards endogenous choice of technology. We prefer rather general than partial equilibrium models of technological choice, having in mind applications to international/interregional trade, where income effects do matter.

Our model describes a closed economy with one homogenous diversified sector (that can be extended to multi-sector economy). It is a classic Dixit-Stiglitz model, but for utility function $u(\cdot)$ being of general (unspecified) form and variable cost $c(\cdot)$ being a decreasing function $c(f)$ of the fixed cost f , that means investment in productivity. Our goal is to find properties of u, c entailing positive or negative impact of the market size on equilibrium prices, outputs, number of firms in the industry, and most importantly - on investment in productivity. Under proper formulation of preferences, these endogenous investments can be interpreted as investment in quality; thereby we simultaneously study productivity and quality.

The findings include complete comparative statics of all equilibrium variables in terms of the Arrow-Pratt measure of concavity, defined for any function g as

$$r_g(z) = -\frac{g''(z)z}{g'(z)}.$$

Our initial intuitive hypothesis was that when facing a bigger population (say, China is bigger than Russia), a firm has more incentives to invest in decreasing variable costs and thus exploit economies of scale. However, actual results below show that such positive correlation holds true only when utility from the varieties displays increasing “relative love for variety” (RLV) r_u and some role is played by $r_{\ln c}$.

Main result. Under reasonable conditions on utility and cost functions (guaranteeing uniqueness of equilibria), the signs of changes in equilibrium variables induced by increasing population size - can be classified into three patterns and some sub-patterns as in Table 1:

	DES		CES	IES		
	$r'_u < 0$		$r'_u = 0$	$r'_u > 0$		
	$r_{\ln c} \leq 1$	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
E_x	$\neg\exists$	-	-	-	0	+
E_{Lx}	$\neg\exists$	-	0	+	+	+
E_f	$\neg\exists$	-	0	+	+	+
E_{Nf}	$\neg\exists$	+	+	+	+	+
E_N	$\neg\exists$	+	+	+	+	?
E_p	$\neg\exists$	+	0	-	-	-
$E_{\frac{p-c}{p}}$	$\neg\exists$	+	0	-	0	+

In Table 1, E_x is the elasticity of individual consumption of each variety with respect to market size (population size), E_f - the elasticity of investment per firm, E_N - the elasticity of mass of firms, E_{Lx} - the elasticity of total output of one variety, E_{Nf} - the elasticity of total investment, E_p - the elasticity of price, $E_{\frac{p-c}{p}}$ - the elasticity of mark-up.

For policy-making our topic may be interesting not only because of new understanding of gains from trade, technological changes in response to trade liberalization and best firm selection. For modernization and active industrial policy practiced in some countries including Russia, it can be interesting, what equilibrium outcome in the sectors of different kind follow from any stimulating measures like tax reductions for R&D. If touching this topic, we should compare social optimum and equilibria in the spirit of Dixit and Stiglitz (1977) and find whether they become closer under governmental stimulation.

Coopetitive Games and Applications

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Keywords: *Games and Economics, Normal form two person games, Cooperative interaction, Coopetitive Games, Macroeconomic, green economy, energy-saving technologies, policy of climate change, Policy, Bargaining solutions*

In this paper (based on researches conducted for three years) we define and apply the new model of coopetitive game (in the sense recently introduced by David Carfi) to Economic Policy, Green Economy and Financial issues and in particular to the crisis of the Euro Zone (as already done in some published articles). The Crisis within the Euro Area has become frequent during 2010 and 2011. First was the Greek economy to face a default problem of its sovereign debt, then it was Ireland who has been in a serious financial situation at the verge of collapse causing difficulties to the euro. In this contribution we focus on the Greek crisis and we suggest, through a coopetitive game model conceived at a macro level, feasible solutions in a cooperative perspective, taking account of the divergent interests that drive the economic policies in Germany and Greece. We conduct a deep study of the particular model proposed, namely, for the analysis we conduct a Complete Analysis of the coopetitive game - in the sense introduced and already applied by D. Carfi in several papers. The key points of our coopetitive exam are essentially the following ones:

- 1) the complete study of an initial game $G(0)$, in the Carfi's sense, from which we obtain also a precise knowledge of its payoff space,
- 2) the study of a curve g of games with starting point the game $G(0)$, by methodologies of essentially geometric nature,
- 3) the determination of the path of Nash equilibria (of the games forming the curve g) (that we will use to the selection of coopetitive Pareto strategies, see point 4),
- 4) the determination of the Pareto maximal boundary of the coopetitive game (that is the maximal boundary of the union of the

payoff spaces of the games forming the curve g),

5) the determination of compromise solutions for our strategic interaction.

From an applicative point of view, our aim is to improve the position of the whole Euro area, also making a contribution to expand the set of macroeconomic policy tools. By means of our general analytical framework of coopetitive game, we show the strategies that could bring to feasible solutions in a cooperative perspective for the different country of the Euro zone (Germany and Greece in particular), where these feasible solutions aim at offering win-win outcomes for all countries in the EMU, letting them to share the pie fairly within a growth path represented by a non-zero sum coopetitive game. A remarkable analytical result of our work consists in the determination of a natural win-win solution by a new coopetitive selection method on the transferable utility Pareto boundary of the coopetitive game.

Moreover the paper proposes a coopetitive model for the Green Economy. It addresses the issue of the climate change policy and the creation and diffusion of low-carbon technologies. In the present paper the complex construct of coopetition is applied at macroeconomic level. The model, based on Game Theory, enables us to offer a set of possible solutions in a coopetitive context, allowing us to find a Pareto solution in a win-win scenario. The model, which is based on the assumption that each country produces a level of output which is determined in a non-cooperative game of Cournot-type and that considers at the same time a coopetitive strategy regarding the low technologies will suggest a solution that show the convenience for each country to participate actively to a program of low carbon technologies within a coopetitive framework to address a policy of climate change, thus aiming at balancing the environmental imbalances.

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Game-Theoretic Formulation of Complex Systems

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Keywords: *Nature, complex system, Pure-strategy, Mixed-strategy, Organization, Principle of Least Action, Nash-equilibrium, Attractor.*

A game-theoretic formulation of complex system is being proposed in this paper. A natural process is a game that tends to organize the system in the least possible time. The Principle of Least Action is applied to formulate the trajectories of the system elements in space. The pay-off for the game is the amount by which the system organizes through various cycles of evolution and optimization. The Nash-equilibrium strategy is being proposed as the path of maximal organization for a system element and that, the existence of the Nash-equilibrium strategy an attractor for all natural systems.

Evolution is a natural process. A natural process is the one that tends to maximize the amount of organization present in a system. Nature essentially consists of complex systems. All systems found in nature are thermodynamically open and physically adaptive. A natural process is defined as an act, by which a system organizes itself with time. Any natural process drives a system to a state of greater organization. Organization is a progressive change that a system constantly encounters while interacting with its surrounding media. The Principle of Least Action has evolved and established itself as the most basic law of physics. This allows us to see how this fundamental law of nature determines the development of the system towards states with less action, i.e., organized states. The amount of action and organization present in a system are the virtue of the constituting elements and constraints (internal constraints to motion of the system elements and the external physical boundary). Greater organized systems tend to be more stable in nature than systems with greater action. Thus, organization and action in any system are related inversely. We can model the evolution of a system hence, a natural-process, as a game-theoretic phenomenon. The intrinsic

properties of complex systems are investigated in this paper. A system undergoing a natural process is formulated as a game that tends to organize the system in the least possible time.

Each of the system elements performs work on the constraints and tends to modify them in order to achieve a state of least action. Presence of too many elements causes each element to behave as a constraint for other elements. Thus, the trajectory of each element not necessarily reduces action but, may cause the system to become highly unstable. Stability and least action state are thus achieved by the system after a number of cycles of evolution. In order to visualize organization in a qualitative way, each system element tends to possess a set of pure strategies and a probability distribution function that maps the set of pure strategies into mixed strategies. The set of pure-strategies for a system element represents the ratio of its actual trajectory of evolution in presence of constraints to the shortest possible path at that instant hence, its Nash-equilibrium path. This is a zero-sum game as each of the elements tends to apply its Nash-equilibrium strategy in order to organize in the least possible time. The pay-off for this game is the amount of organization the system inherits due to the evolution of its constituting elements. The amount by which a system organizes itself is quantified as the inverse of the pure strategies.

Thus, existence of a maximum organized state for a system in nature is virtually impossible. Hence, through various cycles of evolution and self-optimization, the system elements tend to reduce their action by progressively approaching towards their Nash-equilibrium paths. Nash-equilibrium thus, acts as an attractor for all natural systems.

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Game Theoretic Modeling of Corruption in Hierarchical Dynamic Control Systems

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Keywords: *Dynamic hierarchical games, Corruption, Feedback.*

Dynamic game theoretic models of corruption in hierarchical control systems based on the concept of sustainable management ([1]) are considered. These models are a logical sequel of the static game theoretic modeling of corruption in hierarchical control systems ([2]). In the static case, we get a genetic sequence of models: optimization problem (for a bribe-giver) - two-person hierarchical game (bribe-taker - bribe-giver) - three-person hierarchical game (bribe-fighter - bribe-taker - bribe-giver). Optimization model setting corresponds to the case when a bribery function is assumed to be known (a descriptive approach). In the game theoretic settings the bribery function is built as an optimal strategy of the leader (a normative approach). Static game theoretic hierarchical models can contain or not a feedback on control parameters. In the latter case (known as Γ_1 game) we get the Stackelberg equilibrium as a solution concept, in the former case (Γ_2 game) the Germeyer principle of maximal secured payoff is used. Only models with a feedback provide a description of corruption and give a solution of the game based on Germeyer theorem about the Γ_2 game ([2]).

Similarly, in the dynamic case a genetic sequence of models arise: optimal control problem - two-level dynamic hierarchical game - three-level dynamic hierarchical game. In dynamic hierarchical games a different information structure is possible: program strategies (open-loop, Γ_{1t} game) or positional strategies (feedback, memoryless, closed-loop, Γ_{2x} game) ([3]). As a counterpart of the static case, only games in positional strategies can model corruption. Moreover, in the traditional settings

of dynamic hierarchical game theoretic models a positional strategy means a dependence of the control variables of players on the state of controlled system. In the proposed approach corruption is treated as a dependence of the control variables of the leader on the control variables of the follower (Γ_{2t} game). More complex settings with two types of dependence, both on the control variables of the follower and on the state of controlled system are also possible (Γ_{2tx} game). The latter case corresponds to the model of corruption with consideration of the requirements of sustainable development of the controlled system.

In turn, two cases are differentiated according to the methods of control used by the leader. In the case of compulsion, a compulsion control variable of the leader which affects to the set of feasible strategies of the follower depends on bribe (administrative corruption). In the case of impulsion, an impulsion control variable of the leader which affects to the payoff function of the follower depends on bribe (economic corruption).

A dynamic generalization of the Garmyeyer theorem for the Γ_{2t} game is analyzed. Methods of solution of the dynamic games in discrete form with a dependence of the leader control variables on the follower control variable (treated as a bribe) are proposed. Some applications to the optimal exploitation of biological resources and control of water resources quality in the presence of corruption are considered.

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On a Model of informational Control in Active Network Structures

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Keywords: *Network, game theory, Markov model, Information control, Social networks.*

In the last decade interest in the study of network structures has considerably increased, in particular - to the problems of information control in active network structures [4]. First, the active network structures, as opposed to hierarchical, there is no subordination of one element by another. This significantly limits the ability of the investigated in detail the models and mechanisms in the control hierarchy (eg, organizational) structures. Second, the widespread development of information technology and, in particular, the Internet and various online networks (social networks, forums, blogs) has significantly changed and continues to change the environment of social and economic interaction. What will these changes is difficult to say right now, although researchers are trying to make predictions in the economy and policies [1]. We also note that a separate section on network structures, apparently, in the near future it will be in the textbooks on economic theory [3]. In this paper we consider a model of information control in a network structure that operates in accordance with the Markov model, under conditions of incomplete knowledge of the governing person or organization (the principal).

We describe the agents within the network structure (we also call it simply a network), set $N = \{1, 2, \dots, n\}$ Agents in the network affect each other, and the degree of influence given by the matrix A direct effect of dimension $n \times n$ where $\alpha_{ij} > 0$ denotes the degree of confidence in the i -th agent's j -th agent. Here and later we'll talk about how the impact and the confidence and believe that these two concepts are oppositely in

the sense that the term "trust the i -th j -th agent is α_{ij} » is identical in meaning to the expression "degree influence of the j -th agent at the i -th is α_{ij} ».

Let each agent in a certain initial time has some awareness (or, in other words, the opinion) on certain issues. Opinion of i -th agent represents a real number x_i^0 , opinions of all the agents are represented by a column vector x_0 of dimension n . In accordance with a Markov model of agents in the network interacts exchanging opinions. This exchange leads to the opinion of each agent changes in accordance with the opinions of agents to which the agent trusts. Suppose a governing body - the principal - is seeking to get a maximum total value of the characteristics of the agents $\sum_i x_i^\infty$. For more details please see [2]. Let's suppose further that he is able to provide the agents at the initial time control actions u_i , changing their characteristics. Then the total change in the final characteristics of the agents is (here $u = (u_1, \dots, u_n)$) :

$$F = \sum_{j \in N} (A^\infty u)_j = \sum_{j \in N} \left(\sum_{i \in N} a_{ij}^\infty \right) u_j = \sum_{j \in N} w_j u_j.$$

where w_i are some coefficients called influences of agents. Function $F(u)$ is a utility function of the principal, which he seeks to maximize. Having limited resources (for example, if only k components of vector could be different from zero and k less the dimension of vector) should be focused on agents with greater influence. This will give him greater final payoff.

We consider (in different variants of information which the principal has) the following situation: the principal can provide the control action, equal to 1, exactly on k agents (where $k < n$). We are interested in the following two questions:

What is the optimal control strategy of the principal;

What are the networks for the principal of the most and the least profitable?

Proposition 1. In the case of perfect awareness, situation in which the influence of sequence no more than k agents are different from zero is the most profitable for the principal.

Proposition 2. In the case of perfect awareness, situation in which influences of all agents are identical is the least profitable for the principal

Let's consider the network structure, whose dynamics is given by a Markov model but in contrast to previous considerations of the preceding section, we'll assume

that the principal does not know about the influence of specific agents, so influence to each of chosen agents has the same effect equal to 1 for the k randomly chosen agents of n (where $1 < n$). As before, the principal is interested in maximizing the amount of the final characteristics of the agents.

In this case of the uninformed principal of magnitude of his influence on agents $u_i, i=1, \dots, n$ are random. Each of them takes the values from 0 to 1, while $\sum_i u_i = k$

(Unconditional) probability of event $u_i = 1$ is k/n , as well as the (unconditional) the expectation:

$$p(u_i = 1) = E u_i = \frac{k}{n}.$$

The usefulness of the principal is a random variable:

$$F = \sum_{j \in N} w_j u_j.$$

Proposition 3. In the case of the uninformed principal a situation in which the influences of all agents are identical is more profitable for it and situations, when the network structure has a unique element that has non-zero influence are the least favorable.

Now we consider the situation of the partial informed principal: network structure is divided into disjoint subsets, disjoint information, within each of which the principal does not distinguish between agents, but for each subset knows the number of agents and their total influence. In this case strategy of the principal is the choice of the volume impact of the information on each subset, i.e. in which he turns out to be the control action. Thus the total number of agents still to be equal to k , and each k_i , of course, does not exceed the number of agents in the information subset G_i

Proposition 4. In the case of partial awareness of the principal a situation in which the total number of agents in the subset of information subsets, the average influence is that non-zero and do not exceed k is the most profitable for the principal.

Proposition 5. In the case of partial awareness of the principal a situation in which the average influence of all subsets of information islands is the same is the least profitable for the principal

In this paper we consider a model of information control in active network structure under different assumptions about knowledge of the principal. A promising

direction for further research is to analyze the various options and their impact on the effectiveness of information control in active network structures.

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ВЕСТНИК САНКТ-ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

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Selfish Base Station Selection Problem

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Keywords: *Wireless network, Selfish routing, Signal/noise ratio*

The problem of selfish base station selection in wireless network is considered. The problem is presented as a game, where players are mobile connection devices, which choose radio stations to connect to the network. Strategies in the game are probabilities, which players use to choose stations. Players act selfish. Each device chooses a station trying to maximize its ratio "signal/noise" (SNR), which depends on: 1) the distance between the device and the station, 2) connections of all devices on the station. In this model

$$Signal = \frac{1}{(distance_to_BS)^2}, \quad Noise = Signal + \langle Others' _signals \rangle + c.$$

The 1-dimensional game is considered. n base stations have coordinates $b_1 < \dots < b_n$. The players are distributed with a density $p(\cdot)$ on some interval, including $[b_1, b_n]$. Each player knows his own coordinate x , players' number N and the density $p(\cdot)$. Choosing a station players use a rule: when a player in position x is between stations b_i and b_{i+1} he connects to b_i if $x < \tau_i \in [b_i, b_{i+1}]$ and he connects to b_{i+1} otherwise. The target is to determine optimal τ values i.e. moments of switching from one station to another.

The Advantages of Underestimating the Competition

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Keywords: *Bounded rationality, Dynamic games, Bias, Underestimation, Marketing*

Managers repeatedly make business decisions that depend, at least in part, on what they believe their competitors will do in response. Traditional game theoretic models assume that all players are perfectly skilled and given the information available and the rules of the game, all players perfectly optimize their strategies. In other words, these model assume that players have unlimited cognitive capacity and that the process of calculating the optimal strategy is costless.

In the real world, however, people are not capable of infinite calculation, but instead are bounded in the amount of information they can process and causally think through. In complex strategic games, this may mean that players cannot fully optimize their strategies. Furthermore, managers are heterogeneous in their skill levels. Some are “smarter” (more skilled) than others. This heterogeneity creates uncertainty when one player tries to predict what another player will do when faced with a decision set.

These uncertain judgments about the other player’s skill are, of course susceptible to error. Behavioral research suggests that the error is not purely random, but that there is, in fact, a widespread bias towards underestimating others. Conventional wisdom positions this tendency as a costly flaw to overcome, with adages such as “never underestimate the competition,” abounding in popular managerial literature. However, there is a growing stream of research in psychology and bounded rationality that supports the idea that some cognitive biases and seeming irrationalities have evolved as built-in heuristics that simplify decisions and in doing so provide a net long-term advantage when decision effort is limited and costly and environments are complex.

To explore the potential benefits of the observed bias towards underestimating one’s opponent, we build a theoretical model that compares the long-run effects of

consistent underestimation bias with those of overestimation and unbiased random error. We define a structure of extensive form game trees with parameters for both length and complexity, and a continuous distribution from which payoffs at states in the tree are randomly drawn. We define skill by the number of levels of the tree a player can “think through” at any point in time, and create a parameter for the skill of each player. For any specific game, a modified backward induction algorithm can determine the precise payoff outcome for a player that accurately estimates his opponent’s skill, underestimates it by one level, or overestimates it by one level. Building off the distributions used to randomly define payoffs in the game tree, we calculate the expected value of the payoff for each bias type as a function of game structure and player skill parameters. This expected payoff is used to compare the long-term payoffs expected when a player of a particular exogenous skill capacity consistently applies each type of opponent estimation bias when engaging repeatedly in numerous different games (for example, a marketing manager that makes many different decisions involving anticipation of competitive response over the course of an entire career).

We find that, when decision effort (based on the number of possible game paths considered by a player) is at all costly, overestimation is always strictly dominated by the other strategies, and that there exist critical values of several contextual parameters above which underestimation is the strictly dominant strategy. The net advantages of underestimation increase in highly complex environments as measured in terms of the number of moves and the number of options at any given move, and when the players involved are highly skilled, i.e., can see out a large number of moves, and when future payoffs highly correlate with past payoffs, i.e. a rich-get-richer or first-mover-advantage environment. The results support the idea that the widely observed bias towards underestimation may provide overall advantages to some decision-makers, and call into question the conventional wisdom that urges managers to potentially over-invest in researching and predicting competitive response.

Queue Penalization Effect in a Location-Allocation Problem

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Keywords: *Location-allocation problem, leader-follower model, continuous modeling.*

Consider a distribution of citizens in an urban area in which a given number of services must be located. Citizens are partitioned in service regions such that each facility serves the customer demand in one of the service regions. For a fixed location of all the services, every citizen chooses the service minimizing the total cost, i.e. the capacity acquisition cost plus the distribution cost (depending on the travel distance).

In our model there is also a fixed cost of each service depending on its location and an additional cost due to time spent in the queue of a service (depending on the amount of people waiting at the service and on the characteristics of the service - for example, its dimension -). The objective is to find the optimal location of the services in the urban area and the related customers partition.

We present a two-stage optimization model to solve this location-allocation problem. The social planner minimizes the social costs, i.e. the fixed costs plus the waiting time costs, taking into account that the citizens are partitioned in the region according to minimizing the capacity costs plus the distribution costs in the service regions.

Existence results of solutions to the location-allocation problem and computational aspects will be discussed.

Game of Land Acquisition

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Keywords: *Slum Equilibriums, Subgame Perfect, Eminent Domain.*

In the Land economics literature, broadly two classes of models, Bayesian and non-Bayesian are often used. These are either competitive bidding models or urban renewal models. The process by which houses, land, and other forms of real property are sold has been the subject of considerable economic research over the last few decades. However, recent crash in housing prices in US and in Western European countries made the study of this subject even more important. In the Indian context, land acquisition by a Marxist Government in Singuir & Nandigram in West Bengal for Tata Auto Company for setting up a plant for production of as small Nano Car created a great deal of controversy leading to end of 35 year Marxist rule.

The underlying models of land or housing markets can be traced back to the pioneering study of agricultural land-use by von Thünen (1826), but a useful analysis of property value and transaction begins with the classic bid-rent models of Alonso (1964), Muth (1968), Beckmann (1969), Montesano (1972), Wheaton (1974), and Hartwick et al, (1976). Horowitz (1988) criticizes classic bid-rent approach for unrealistically assuming market perfection. Recently, Kaneko (1983), Gerber (1985), Eckart (1985), and Asami (1988), Flaherty (1988), Asami and Teraki (1990), Deman and Wen (1994), and Deman (2000) have developed a few game theoretic models to address the problem of site assembly subject to indivisibility. However, with the exception of Eckart (1985), Deman and Wen (1994), and Deman (2000), all other models ignored very important economic realities: rational expectations and strategic interactions between the players. Furthermore, none of these models address issues related to market efficiency, which is crucial for studying real estate takeovers.

In this paper I use a subgame perfect approach to the problem of a State or its agents who want to acquire land for Industrial Development. The problem outlined in this paper is a combination of the problems addressed by Grossman and Hart (1980) and by Shleifer and Vishny (1986) in the corporate control literature. The relevant corporate story of Shleifer and Vishny (1986) concerns value-improving monitoring performed by others.

The basic distinction between Grossman & Hart above paper and the others is the assumption of a continuum of players and atomistic agents. Why should this matter? I will elaborate this briefly. With a continuum of players' assumption, the paradox is caused by the "disappearance of information" because only the aggregate play is observed. Hence, the individual deviation cannot be met by rewards and punishments. In a finite number of players game, there is a change in the aggregate play whenever a player deviates. The change may be very small, but perfectly observable. Therefore deviations can be rewarded or punished regardless of number of players. Hence, the results are straight forward that the acquisition will occur with positive probability in models with finitely many players.

My research not only provides a useful application of Grossman & Hart (1980) theory but also extends their theory on takeovers. Using measure theory of statistics, I provide a formal mathematical structure to Land development model. I look at a class of games to include both, the continuum and finitely many homeowners to contrast the situation when the redevelopment project is indivisible and partially divisible. In pure strategy equilibrium a redevelopment projects require all land in the target to be economically feasible which can be realised through purchase by secrecy.

I assume many incumbent land owners are basically homogeneous. However, many urban renewal projects do not match these characteristics. Inner city residential redevelopment, for example, usually takes place in deteriorated existing neighbourhoods, and does not require a specific scale so long as it exceeds a threshold level. The relaxation of complete indivisibility also introduces variation to land owners' bargaining power, which cannot be handled by Gross & Hart (1980) model with uniform dilution. The richness of the model is further enhancement by the introduction of strategic interaction and rational expectations character into the model. Unlike Grossman & Hart model I also show that one cannot completely rule out a possibility of the takeover by an inefficient developer, which may result in an inefficient Nash equilibrium.

The story in the land acquisition framework is further complicated by the fact these land owners may differ in their valuation of even otherwise identical units since the owners may have different attachments to their land. In my paper I introduce the notion of dilution as the participating government agency's ability to force land owners to sell to the developer or comply with costly new standards for development. Dilution increases the costs to potential holdouts. I follow Milgrom and Roberts (1982) paper on Limit Pricing and Entry and use a subgame perfect approach under complete but imperfect information in the game.

At the policy level my research intends to develop a corporate finance-game theoretic model to describe a broad range of land development projects. It can also help in predicting the outcomes of land takeover by developers, and evaluate the effectiveness and appropriateness of their using dilution (a privatised eminent domain in the context of land acquisition) in the process. In the absence of direct public intervention, land development is in essence private investment in public goods. While Pareto improvement could be achieved by such undertaking, the "unexcludability" of public goods tends to provide incumbent property owners incentives of free riding on the improvements, and results in "holdout" problems that often hinder successful redevelopment. Certainly the government could step in whenever holdouts occur and exercise its power of eminent domains to wipe them out, but in so doing it will invite scepticism about its impartiality as it appeared in Nandigram and Singur. Leaving alone public intervention, could we solve the holdout problem by market mechanisms? To this date, no satisfactory answer has been offered in the urban or land economics literature. My paper, in fact, just does that and also addresses some of the above questions. The paper also provides an analytical framework for studying a broad range of privatised land development by organising sales of land at par with sales of stocks for a corporation.

The main conclusion of the paper is that the threat of takeover can facilitate redevelopment programme undertaken by a developer even though the equilibria are straightforward in a finitely many players' game for the reasons stated above. I also show that the developer faces a trade-off between a low bid price and a high probability of takeover. However, the probability of takeover can be enhanced by the provision of dilution or exercise of right to eminent domain. Paper makes a useful application of game theory and corporate finance to address policy issues of dilution, imminent domain, slum equilibria, and other urban renewal problems. Paper also provides

evidence in favour of how the market for development purchases should be organised. In fact, it seems like a potentially interesting idea and it might even be possible for a local government to adopt such a rule. To minimise the government abuse of right of eminent domain and to enhance the effectiveness of using eminent domain as a threat by the developer I introduce a democratic (50% yes vote) or super majority rule in the model (to pass a constitutional test).

Results are significantly different from those already published and will appeal to a wider audience from finance, industrial organisation, game theory and urban economics. It would also contribute to ongoing debate on the subject of takeovers. I believe the limiting case of the finite owner model-as the number of owners-tend to infinity-differ from continuum case. However, it can be shown that a finitely many player model with a noise will give the same results as the continuum of players model. A generalization of such a model is not difficult, but preliminary results may be usefulness in formulating public policies regarding privatisation of urban renewal efforts.



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GAMES AND ECONOMIC BEHAVIOR

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Capacity market impact on investment incentives

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Keywords: *Capacity market, Subgame perfect equilibrium, Discriminatory auction, Investment incentives*

Capacity market models were initially designed as an instrument to increase investment incentives for electricity suppliers. The main idea of these markets is to force and make it profitable for firms (suppliers) to build as many capacities of the needed type and quality on the defined territories as the whole power system requires. Among the most important challenges facing the capacity market in Russia were:

- ensuring investment incentives: attracting investment in new electrical capacities and infrastructure;

- supporting the firms' current electricity capacities: compensation of fixed costs by revenue from sale of capacity, as revenue from sale of electricity is limited by the price-cap and thus insufficient.

The present paper aims to study a game-theoretic model of electricity capacity market. We show the existence of subgame perfect equilibria, their form, answer the questions of multiplicity of equilibria depending on values of model parameters.

Kreps, Scheinkman (1983) proposed a two stage game in which capacity decisions made prior to spot electricity market, price competition stage. We consider the similar 'Bertrand-Edgeworth' type model when consumers pay the price of the firm from which they are supplied.

Fabra, von der Fehr, Harbord (2006) and Fabra, von der Fehr (2008) consider a symmetric duopoly with zero marginal costs and a two stage market with price-cap. Authors analyzed a number of different market design elements, including the uniform-price and discriminatory auctions, price-caps and bid duration (firms compete in prices prior to demand realization). They found that, although the discriminatory auction tended to lower prices, this did not imply that investment incentives at the margin were

poorer; under reasonable assumptions on the shape of the demand distribution, the discriminatory auction induced (weakly) stronger investment incentives than the uniform-price format.

We consider a symmetric duopoly with cost function $C^i(q)$, where $i = \{1, 2\}$, constant marginal costs mc and cost per unit of capacity c . Demand function is inelastic and follows a binomial distribution function: low demand C^l occurs with probability ρ , and high demand D^h - with probability $1 - \rho$, consider $0 < D^l < D^h$.

On the first stage of the game, firms make investment decisions $q_i, i = 1, 2$ simultaneously with demand uncertainty. Once investment decisions have been made, information about capacities q_1, q_2 becomes a public knowledge. Next demand is realized and publicly observed and firms compete in prices in discriminatory auction by choosing $p_i, i = 1, 2$ - the lowest price they want to sell electricity. The volume of electricity sold cannot be larger than invested capacities $q_i, i = 1, 2$. Each firm bid maximizes firm's profit and the bid profile is found as a subgame perfect equilibrium. Price bids cannot exceed the price-cap r ($p_i \leq r, i = 1, 2$), such that $r - mc > c$. Firms supply the demand ordered in increasing of their price bid p_i .

Proposition 1: Suppose that the forecasted demand is D and firms made investment decisions $q_i, i = 1, 2$, then the equilibrium price strategies are characterized as the following::

A) For $q_1 \geq D$ there exists a unique pure-strategy equilibrium with prices set equal to the marginal cost $p_1^* = p_2^* = mc$. Expected profits of the firms are equal and are zero: $Pr_1^*(p_1, p_2) = Pr_2^*(p_1, p_2) = 0$.

B) For $q_1 < D < q_1 + q_2$ there does not exist a pure-strategy equilibrium. There exists a unique mixed-strategy equilibrium with the equilibrium prices in the interval:

$(p_1^*, p_2^*) \in [mc + \frac{(r - mc)(D - q_1)}{\min(q_2, D)}, r]$ and equilibrium price distributions for the small

firm and the large firm respectively:

$$F_1^*(p_1) = \frac{(r - mc)[\min(q_2, D) + q_1 - D] + q_1(p_1 - r)}{(p_1 - mc)[\min(q_2, D) + q_1 - D]}$$

$$F_2^*(p_1) = \frac{(r - mc)[\min(q_2, D) + q_1 - D] + \min(q_2, D)(p_2 - r)}{(p_2 - mc)[\min(q_2, D) + q_1 - D]}$$

And firms' profit functions defined as following:

$$Pr_1^*(p_1, p_2) = \frac{(r - mc)(D - q_1)q_1}{\min(q_2, D)}$$

$$Pr_2^*(p_1, p_2) = (r - mc)(D - q_1)$$

C) For $q_1 + q_2 \leq D$ there exists a unique pure-strategy equilibrium with prices set equal to the price-cap: $p_1^* = p_2^* = r$. In this case the firms sell all their capacities and have profit functions:

$$Pr_1^*(p_1, p_2) = (r - mc)q_1$$

$$Pr_2^*(p_1, p_2) = (r - mc)q_2$$

Proposition 2: Suppose that the demand can take either the value D^l with probability ρ , or the value D^h with probability $1 - \rho$, where $\rho \in [0, 1]$ and $D^l < D^h$. Then there exist a continuum of subgame perfect pure strategy equilibria such that, for $\rho \in [0, 1 - \frac{c}{r - mc}]$ the aggregate capacity of firms is equal to the highest demand value:

$q_1 + q_2 = D^h$, for $\rho \in [1 - \frac{c}{r - mc}, 1]$ the aggregate capacity is equal to the lowest demand

value: $q_1 + q_2 = D^l$, for $\rho = 1 - \frac{c}{r - mc}$ any aggregate capacity in the interval

$q_1 + q_2 = [D^l, D^h]$ can be sustained as an equilibrium.

Proposition 3: If the forecasted demand is more likely takes the maximum value ($\rho < 1 - \frac{c}{r - mc}$), then the profit per unit of capacity of the small firm is larger or equal to the one of the large firm.

Proposition 4: There exist symmetric subgame perfect equilibria for any market structure except for

$$\rho \in \left(\left(\frac{1}{2} - \frac{c}{2(r - mc)} \right) \left(1 + \sqrt{1 - \frac{2}{\Delta}} \right), 1 - \frac{c}{r - mc} \right), \text{ where } \Delta = \frac{D_h}{D_l}.$$

We enhanced results of Fabra, von der Fehr having focused on binomial distribution of demand function: considered model with nonzero marginal cost; gave full characteristics of equilibrium strategies for price competition stage (the set of

equilibrium price values, equilibrium price distribution functions) and capacity choices depending on the values of market parameters.

Obtained results demonstrate that the market architecture with the price competition stage and demand uncertainty on the investment decision stage leads to asymmetry in the profits per unit of capacity. This results in the asymmetry of investment incentives. Although firms are initially symmetric, the set of subgame perfect equilibrium appears to be asymmetric with large and small firms in the market. The firm with smaller capacity bids more aggressively than the firm with larger capacity as its capacity deviation leads to major change in its profit.

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Strategic Experimentation in R&D Races

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Keywords: *Strategic experimentation, Two-armed bandit model, Inefficiencies*

R&D competition refers to situations where several firms race to be the first to create an invention or, in certain cases, gain a lasting technological lead. In such a situation, information on the advancement of a rival firm is a crucial element in the investment decision. For example, a firm may abandon a research project or instead double its efforts, depending on whether a rival has a significant technological lead, an important cost advantage in a certain type of R&D activity, or a scientific advantage due to its other research projects.

Learning that a rival firm made a breakthrough and is ahead has two opposite effects on the lagging firm. The R&D literature identifies the negative aspect, which is the strategic effect of being behind, while the experimentation literature identifies the positive aspect which is the technological feasibility of making a breakthrough. Thus, while the current R&D literature identifies the inefficiency involved in over-investment as a result of externalities ignored by the lagging firm, the experimentation literature identifies the fundamental inefficiency of information acquisition because of free-riding. This paper is a first attempt to unify these two strands of the literature. Indeed, in our model both of these opposite forces are at play. A central result that we establish is that the free riding effect, and hence under-investment, is the dominating one in the first stages of the race while over-investment is the more important effect in the more advanced stages of the race.

The importance of information about a rival's breakthrough, and its effect on the race, is illustrated in the example of the race to complete the sequencing of human genes between The Human Genome Project, a publicly funded consortium of university labs

located mainly in the United States and Britain, and Celera Genomics, a private firm based in Maryland. The Human Genome Project started in 1990, with a target completion date of 2005. The work proceeded slowly however until 1998, when after completing the sequencing of the Bacterium *Haemophilus Influenzae* in 1995, it became known that Craig Venter, founder of Celera, had obtained private funds to form a company that would sequence the genome by 2001.

The immediate reaction was mixed among the researchers and sponsors of the Human Genome Project. The Wellcome Trust, a British institution that sponsored the British labs, reacted by substantially increasing the funding, while U.S. authorities tergiversated for some time. A reporter of the New York Times who covered the entire race commented: [The U.S.] Congress might ask why it should continue to finance the Human Genome Project through the National Institute of Health and the Department of Energy if [Celera] is going to finish first.

In the following years, Celera issued a series of press releases announcing successes. In September 1999, it announced the sequencing of the fruit fly genome in five months. In comparison, it had taken Sulston's team nine years to sequence the worm *Caenorhabditis elegans*. According to Sulston and Ferry (2002), a relentless barrage of Celera press releases made it look as though they were simply blowing the public project out of the water. [...] Celera lost no opportunity to make unfavorable comparisons with "other early genomes" that had, in the words of the press statement, taken "over a decade" to complete.

To study this phenomenon, we analyze a game of strategic R&D race based upon two-armed bandit technologies. The race is between two firms, which must each complete two phases of research, an initial phase of basic research and a final phase of development. Phases capture the idea of progress or position in a research program. The winner of the race is the firm which completes the final phase first. Both firms start in the initial phase using a similar technology, which is either good for the research purpose and generates a positive probability of completing the initial phase and advancing to the final phase, or it is bad and investing in it is a complete waste. In every phase the firms have replica two-armed bandit technologies with all risky arms being of the same type (all good, or all bad), but with "breakthroughs" occurring independently. In every period each firm has to decide whether or not to spend resources on R&D (pulling the risky arm). Firms observe each others' actions and outcomes, so information about the type of the risky arm is a public good.

Such a game of strategic experimentation is typical among firms involved in R&D races where, on one hand, given the flow of information, they will draw on each others' experience, and thus have an incentive to postpone investment and free ride on one's rival, but on the other hand allowing one's rival to be the first to make progress in the race puts one in a disadvantage. The model studied here captures this trade off. When both firms are in the initial phase then they each draw on each others' experiences, and a lack of success would make all parties more pessimistic. The first firm to make a breakthrough reveals that the state is good. A breakthrough by one firm, is, on one hand, a good news to the rival, as it is now common knowledge that advancing to the final phase is feasible, but on the other hand, this is bad news to the rival who is now lagging in the race for the prize.

Thus, while current R&D literature identifies the inefficiency involved in over investment as a result of externalities ignored by the firm, and the experimentation literature identify the fundamental inefficiency of information acquisition because of free-riding, in our model both of these forces are at play.



Journals in Game Theory

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SPRINGER



Playability properties in games of deterrence and evolution in the Replicator Dynamics

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Keywords: *Evolutionary games, Fuzzy logic, Games of deterrence, Graphs of deterrence, Playability, Replicator Dynamics, Species, Strategies*

Since the seminal work of John Maynard Smith [3], a vast literature has developed on evolution analysis through game theoretic tools. Among the most popular evolutionary systems is the Replicator Dynamics, based in its classical version on the combination between a standard non cooperative matrix game and a dynamic system which evolution depends on the payoffs of the interacting species. Despite its weaknesses, in particular the fact that it does not take into account emergence and development of species that did not initially exist, the Replicator Dynamics has the advantage of proposing a quite simple model that analyzes and tests some core features of Darwinian evolution.

Nevertheless, the simplicity of the model reaches its limits when one needs to predict with high precision the conditions for reaching evolutionary stability. The reason for it is quite obvious: it stems from the difficulties to find an analytical solution to the system of equations modelling the Replicator Dynamics.

An alternative approach has been developed, based on a different kind of matrix games, called games of deterrence. Matrix games of deterrence are qualitative binary games in which selection of strategic pairs result for each player in only two possible outcomes: acceptable (noted 1) and unacceptable (noted 0). It has been shown (Rudnianski [4]) that each matrix game of deterrence can be associated in a one to one relation with a system of equations called the playability system, the solutions of which

determine the playability properties of the players' strategies. Ellison & Rudnianski have shown [1] that one could derive evolutionary stability properties of the Replicator Dynamics from the solutions of the playability system associated with a symmetric matrix game of deterrence on which the Replicator Dynamics is based.

Thus, it has been established that to each symmetric solution of the playability system corresponds an evolutionarily stable equilibrium set (ESES). Also if a strategy is not playable in every solution of the playability system, the proportion of the corresponding species in the Replicator Dynamics vanishes with time in every solution of the dynamic system.

Based on these results, the proposed paper has two objectives. The first one is to extend the analysis already undertaken and propose new results in terms of relations between the solutions of the game of deterrence playability system and the solutions of the dynamic system. The second objective is to develop a method that will extend the above approach, initially restricted to a Replicator Dynamics associated with a game of deterrence to a Replicator Dynamics associated with a quantitative symmetric matrix game, through showing that one can associate with such games, a game of deterrence such that some of the evolution properties associated with the games of deterrence are still verified when the latter are replaced by quantitative games.

More precisely, in a first part, after having briefly recalled the definition of the Replicator Dynamics, the paper will recall the definitions and basic properties of games of deterrence. In particular it will be stressed that matrix games of deterrence can be associated in a one to one relation with a specific category of graphs called graphs of deterrence, such that there is a tight relation between the type of graph (path, tree, circuit, etc.) and the solution set of the game of deterrence.

A second part will use the results available on graphs of deterrence, to associate the games of deterrence corresponding to a particular graph of deterrence with a specific solution set of the system representing the Replicator Dynamics.

A third part will then develop an algorithm which will associate a game of deterrence with any standard quantitative symmetric matrix game in a way that will enable to generalize the method to the analysis of quantitative evolutionary games.

A fourth part will analyze the relations between fuzzy solutions of a game of deterrence playability system and the properties of evolutionary games solutions.

On the whole, the proposed paper will significantly improve the capacity to predict the evolution of systems modelled through the Replicator Dynamics.

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Dependence of Nash Equilibria on Incompetence

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There are many application areas for game theory where it is desirable to analyse the behaviour of the game as the game itself changes. For example in the training of sports people, we can examine how the strategies and payoffs for the player change as their skills increase. Also, in new equipment purchase for defence forces, a key problem is to examine how payoffs and strategies change as new capabilities are purchased, often for vast amounts of money.

These problems are being addressed through introducing the notion of incompetence to standard non-cooperative games. Incompetence represents the concept that the action desired by a player might not always be the action that they are able to execute. For instance, tennis players who desire to serve aces may instead serve faults due to their inability to always execute their chosen action. Incompetence is represented by an incompetence matrix which maps how the desired actions translate to actual outcomes. In this presentation we describe how incompetence is introduced into bimatrix games and demonstrate the results in some simple examples. These examples show that the number of equilibria can change as the level of capability or training changes along with other properties. Identification of critical levels of training where changes occur now emerges as a challenging problem in the theory of games with incompetent players. Larger examples follow to demonstrate some of the more complex behaviours that are possible. Finally, several analytic results are presented.

Does Flexibility Facilitate Sustainability of Cooperation over Time? A Case Study from Environmental Economics

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Keywords: *Incentive equilibrium strategies, Differential games, Environmental economics.*

The use of incentive equilibrium strategies is one of the approaches proposed in the literature to ensure the sustainability over time of an agreement reached at the starting date of a two-player differential game ([2], [3]). The equilibrium incentive approach allows embodying the agreed (cooperative) solution with an equilibrium property. Therefore, by definition each player will find individually rational to stick to his part in the coordinated solution. The incentive strategies are defined as functions of the possible deviation of the other player and recommend to each player to implement his part of the agreement whenever the other player is doing so. These strategies are credible if each player will implement his incentive strategy and not the coordinated solution if he observes that the other one has deviated from the agreement. The credibility of linear incentive strategies for the class of linear-state and linear-quadratic differential games has been studied in [4], [5]. In these papers the authors show that if the analysis is restricted to linear incentive strategies and the special structures of the games are preserved, the linear incentive strategies are not always credible.

The focus of this paper is to show that the definition of more flexible non-linear incentive equilibrium strategies for two-player differential games helps to guarantee the sustainability of the agreement over time. The aim of the study is to check if the definition of less restricted incentive strategies in terms of the permitted deviation from the coordinated solution facilitates the credibility and implementation of these strategies. To this end, we consider a class of incentive strategies that are defined as non-linear functions of the control variables of both players and the current value of the state variable. We show that it is possible to choose the incentive strategy functions in such a

manner that the optimal state path evolves arbitrary close to the corresponding cooperative state trajectory.

We illustrate the use of these strategies on a well-known example drawn from environmental economics, a transboundary pollution differential game [1]. We present some hints of the mathematical analysis. Numerical experiments are presented to illustrate the results. Essentially the numerical algorithm consists on solving an approximate time-discrete dynamic game. The dynamic programming equations are solved by a Chebyshev collocation method.

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Optimal Strategies in a Differential Game with Two Pursuers and One Evader¹

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Keywords: *Differential games, Group pursuit games, Level sets of value function, Switching lines, Optimal controls*

A model differential game with two pursuers and one evader is considered. All three objects move in one line. The dynamics of the pursuers P_1 and P_2 is described by relations

$$\begin{aligned} \ddot{z}_{P_1} &= a_{P_1}, & \ddot{z}_{P_2} &= a_{P_2}, \\ \dot{a}_{P_1} &= (u_1 - a_{P_1}) / l_{P_1}, & \dot{a}_{P_2} &= (u_2 - a_{P_2}) / l_{P_2}, \\ |u_1| &\leq \mu_1, & |u_2| &\leq \mu_2, \\ a_{P_1}(t_0) &= 0, & a_{P_2}(t_0) &= 0. \end{aligned} \tag{1}$$

Here, z_{P_1} и z_{P_2} are the geometric coordinates of the pursuers, a_{P_1} и a_{P_2} are their accelerations generated by the controls u_1 and u_2 . The time constants l_{P_1} and l_{P_2} define how inertially the controls affect the system.

The dynamics of the evader E is analogous:

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$$\begin{aligned} \ddot{z}_E &= a_E, & \dot{a}_E &= (v - a_E) / l_E, \\ |v| &\leq v, & a_E(t_0) &= 0. \end{aligned} \quad (2)$$

Let us fix two instants T_1 and T_2 . At the instant T_1 , we compute the miss of the first pursuer with respect to the evader, and at the instant T_2 , the miss of the second pursuer is computed:

$$r_{P_1,E}(T_1) = |z_E(T_1) - z_{P_1}(T_1)|, \quad r_{P_2,E}(T_2) = |z_E(T_2) - z_{P_2}(T_2)|. \quad (3)$$

Suppose that the pursuers act together. Therefore, we can join them into one player, which governs the vector control $u = (u_1, u_2)^T$. Let us call it as the first player and let the evader E be the second player. The resultant miss is computed as

$$\varphi = \min\{r_{P_1,E}(T_1), r_{P_2,E}(T_2)\}. \quad (4)$$

At any current instant, both players know exactly phase coordinates $z_{P_1}, \dot{z}_{P_1}, a_{P_1}, z_{P_2}, \dot{z}_{P_2}, a_{P_2}, z_E, \dot{z}_E, a_E$. The first player generating his feedback control minimizes the miss φ , the second player maximizes it.

Thus, we consider game (1) — (3) as a standard antagonistic differential game [1]. It can also be considered as a special type game, namely, a group pursuit game [1,2,3,4,5].

The model differential game described above appears [6,7] during study of a situation when two weak maneuvering objects pursuit another one in the horizontal plane. An essential assumptions is that the velocities of nominal motions are quite large and, therefore, the individual misses of each pursuer can be measured at the corresponding instant of the nominal collision.

The main objective of the talk is to show how preliminary construction of level sets of the value function and their procession allows to construct switching lines, which depend on time and define quasioptimal feedback controls u_1, u_2 of the first player and v of the second one. Strict proves are made now only for the case of "strong" pursuers. Results of numeric simulations are given.

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Imitation Based on CG-3x3 Interaction Game in Networks

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Keywords: *CG-3x3 interaction game, Imitation, Strategic interaction, Information neighborhood, The imitate-the-best rule*

In this paper we consider a population of players, who are located around a circle and interact only with their immediate neighbors. However, each player can observe the behavior of (some) other players, who serve as information neighbors. Then all players choose their actions from that behavior by an imitate-the-best rule. It is important that the information neighborhood does not always include the interaction neighborhood in our model, in other words we work with local plus random information. In the context of imitation, we study the relation between imitation outcomes based on CG-3x3 interaction game in a circle network and payoff variables.

On the basis of simulation system Netlogo, we program and illustrate the strategic interaction and imitation of players in network so that we can intuitively process the theoretical derivation in the help of equilibrium results.

Model of mobile service quality choice under competition: Russian market

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Keywords: *Quality choice, Quality inclination, Willingness to pay, Exponential distribution, Two-stage game, Nash equilibrium, Optimal quality differentiation, Russian mobile service market.*

Mobile service market in Russia is one of the most developed and leading industries. In 2000 its market share was 25% and increased to 43% in 2010. According to AC&M Consulting the number of mobile service users in St. Petersburg is 12,93 billion users (31.01.2011) which corresponds to 203,9 % penetration rate. In such market condition strong competition between mobile operators intensifies even more. And while before the competition partook the type of price competition, nowadays mobile operators mend their fences by means of mobile service quality increase. Therefore, for the mobile service market where the level of competition is high (there are at least 3 operators in each Russian region) the problem of mobile service quality choice under competition becomes important and relevant.

In this paper game-theoretical model of quality level choice under competition is suggested in order to define optimal quality strategies of mobile operators. Suppose that five firms provide homogeneous services differentiated by quality on the industrial market. The model is based on the game-theoretical model of quality choice under competition presented in [Tirole, 1988] and its extension in [Aoki, 1996], [Motta, 1997] and [Benassi, 2006].

The game-theoretical model is presented as dynamic game which consist of the following stages: a) each firm i chooses its service quality levels s_i ; b) firms compete in price p_i .

Consumers differ in their willingness to pay for quality level s , which is described by the parameter $\theta \geq 0$. This parameter is called “inclination to quality”. The

utility of a consumer with a willingness to pay for quality θ when buying a service of quality s at a price p is equal to:

$$U_{\theta}(p) = \begin{cases} \theta s - p, & p \leq \theta s \\ 0, & p > \theta s \end{cases}.$$

The investigated industrial market is considered to be partially covered.

In this paper we suggest the model when inclination to quality is exponentially distributed. This means that the majority of consumers have the willingness to buy services with the critical level of quality. In this paper the situation when consumers are eager to buy the lowest level of quality is considered, but it may be extended to the situation with the highest level of quality.

The payoff function of the firm i which provides a service of quality s_i , where $s_i \in [\underline{s}, \bar{s}]$, is the following:

$$R_i(p_1, p_2, s_1, s_2) = p_i(s_1, s_2) D_i(p_1, p_2, s_1, s_2), \quad i = 1, 2,$$

where $p_i(s_1, s_2)$ is the price of the product (or service) of the firm i , $D_i(p_1, p_2, s_1, s_2)$ – the demand function for the product (or service) of quality s_i , which is specified.

The problem of equilibrium estimation is solved using backward induction. The strong Nash equilibrium in the investigated game was obtained in the explicit form which allowed us to evaluate prices, companies' market shares and revenues in the equilibrium.

The survey was conducted in St. Petersburg and defined consumer preferences and satisfaction with mobile service. The survey and game-theoretical analysis of St. Petersburg industrial market allowed finding current and equilibrium service quality levels. The results showed that all operators should increase service quality (table 1).

Table 1. Mobile service quality levels comparison.

	MTS	Megafon	Beeline	Skylink	Tele2
Current state	0,566	0,529	0,479	0,433	0,499
Equilibrium	0,800	0,664	0,608	0,570	0,538

Following the equilibrium quality strategies, mobile operators may achieve the increase of the average consumers' monthly expenses (table 2).

Table 2. Increase in consumers monthly expenses for mobile services.

	MTS	Megafon	Beeline	Skylink	Tele2
Expected monthly expenses	52,50%	26,87%	27,24%	31,67%	7,85%

In the table 3 the results of market shares comparison is presented: first line represents the current market shares according to the survey, while the second line shows how the situation changes when applying game-theoretical results for mobile operators of St.Petersburg.

Table 3. Market shares comparison.

	MTS	Megafon	Beeline	Skylink	Tele2
Current state	28,4%	42,8%	11,0%	3,4%	14,5%
Equilibrium	42,6%	33,1%	16,3%	6,2%	1,7%

On the Equivalence of Bayesian and Dominant Strategy Implementation in a General Class of Social Choice Problems

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Keywords: *Mechanism design, Bayesian implementation, Dominant strategy implementation, Social choice problems*

We consider a standard social choice environment with linear utilities and independent, one-dimensional, private values. We provide a short and constructive proof that for any Bayesian incentive compatible mechanism there exists an equivalent dominant strategy incentive compatible mechanism that delivers the same interim expected utilities for all agents. We demonstrate the usefulness and applicability of our approach with several examples. Finally, we show that the equivalence between Bayesian and dominant strategy implementation generally breaks down when utilities are non-linear or when values are interdependent, multi-dimensional, or correlated.

A Social Capital Index

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Keywords: *Social capital, Centrality, TU game, Shapley value, Myerson value.*

Given a cooperative game and a communication graph we define an index of a player's individual social capital as an excess of the Myerson value over the Shapley value of the player which in turn equals to the Shapley value of the player in the game being the difference between Myerson restricted game of the given game and the given game itself. The consideration of the difference between the Myerson and Shapley values of a player provides a tool for revealing the influence of the player's social network relations to the outcome of the game. Remark that the such defined social capital index is ideologically close to the centrality measure introduced in Gomez et al. (2003). But while in the above mentioned paper the authors define the centrality measure only for evaluation of a player's positional importance in a graph avoiding a priori differences among players and thus using a symmetric game, we define the social capital index as an index of player's relational importance admitting that players possibly have different cooperative abilities.

In the paper we study general properties of the social capital index and reveal its upper and lower bounds. We show that, given a game and a communication graph, the social capital of a player reaches its maximum when the communication graph is a star and the player is the hub of this star, while the social capital of a player is minimal when the player is an isolated point in a communication graph. The computations done for two

real-life examples: (i) 1983 Italian Parliament elections when Bettino Craxi from the Italian Socialist Party, which got in the Lower Chamber only 73 seats from the total amount of 630 but had very strong central position, became the Prime Minister, and (ii) 2009 Basque Country Parliament elections when the Basque Nationalist Party winning the maximal number of seats finally was not included into the majority due to its weak communication ability on the political spectrum, clearly show the coincidence of our theoretical predictions with the reality.



Periodicals in Game Theory

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Model of Supply Network Formation Management

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Keywords: *Network formation game, Principal-agent model, Supply network, Structure optimization.*

A principal managing multiple agents is considered. Agents engage in a network formation game by creating directed links with each other, while the principal pays them to stimulate formation of the desired network. Principal maximizes the difference between revenue derived from the network and rewards paid to the agents, while each agent maximizes the difference between the received reward and costs for links creation and maintenance. The novelty of the setting is that the set of agents is also chosen by the principal.

Suppose the principal chooses some set of agents and wants to implement a certain network as a strong Nash equilibrium of the agents' game. Then, as shown in Gubko (2004), he or she has to compensate each agent's costs of this network formation. This gives rise to a discrete optimization problem of searching a cost-minimizing network over a set of admissible networks.

We study a subclass of optimal network problems originated from the models of supply network design. With the set S of producers and the set D of consumers being fixed along with their demands and supplies $L \subseteq S \times D$, the problem is to build a network of intermediary agents who dispatch and distribute product flows. Cost function $c(\{s_1, \dots, s_k\}, \{d_1, \dots, d_l\})$ of an agent depend on the flows $s_1, \dots, s_k, d_1, \dots, d_l \subseteq L$, running through the input and output links of the agent. This general notation catches both flow routing complexity effects (costs may depend on the number of links) and scale effects (costs may depend on the content of each input or output link, e.g. flows intensity).

Instead of searching a cost-minimizing network for every possible set of agents, we look for the locally tested conditions on a cost function that make possible general

predictions about the shape of an optimal network. These implications limit the search space and release from redundant networks enumeration.

It is established that in the absence of routing complexity effects the optimal structure of the network is pretty simple and consists of no more than two intermediate layers. The conditions on the cost function are derived for each intermediary node of the optimal network to have exactly two input links and/or two output links. These conditions generalize the notion of the, so called, narrowing cost function, introduced by Voronin and Mishin (2002) for the hierarchy optimization problem.

We prove that in the absence of scale effects few technical assumptions result in the optimality of a coupled tree. It consists of a concentrating funnel and a distributing funnel connected by their necks. We show that every node in a concentrating funnel of an optimal coupled tree tends to have the same in-degree, and the same holds for out-degrees of the nodes in a distributing funnel. We use the results of Goubko, Mishin (2008) to derive analytic estimates for the optimal degrees, and also for the costs of an optimal coupled tree.

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Coalitional Model of Decision-Making over the Set of Projects with Different Preferences of Players¹

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Keywords: *Coalitional game, PMS-vector, Compromise solution.*

Let be N the set of players and M the set of projects. The coalitional model of decision-making over the set of projects is formalized as family of games with different fixed coalitional partitions for each project that required the adoption of a positive or negative decision by each of the players. The players' strategies are decisions about each of the project. Players can form coalitions in order to obtain higher income. Thus, for each project a coalitional game is defined. In each coalitional game it is required to find in some sense optimal solution. Solving successively each of the coalitional games, we get the set of optimal n -tuples for all coalitional games. It is required to find a compromise solution for the choice of a project, i. e. it is required to find a compromise coalitional partition. As an optimality principles are accepted generalized PMS-vector [1, 2] and its modifications, and compromise solution [3]. The proposed paper is the generalization of the our paper "Static Model of Decision-making over the Set of Coalitional Partitions" [4] for the case when the preferences of players are different.

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Extended SIR Model: Economic and Social Aspects

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Keywords: *Evolutionary game, Vaccination problem, Optimal control, Epidemic process, SIR model*

This work is focused on the classical SIR model, applied to the epidemic of respiratory infection in total urban population. Total urban population is divided into three groups: S is Susceptible, I is Infected and R is Recovered. We consider two modifications of the classical SIR model:

- The group of Susceptible is vaccinated.
- Total urban population is divided into several risk-groups subject to the structure of the population and connection between individuals in population.

For both modifications we use Gillespie (Gillespie, 1977) algorithm to simulate the epidemic process. As a result we found time interval such as the number of Infected in maximal that corresponds to the epidemic peak. Based on the results of the numerical simulations we extend SIR model and take into account preventive measures that can be applied to group of Susceptible. Preventive measures help to avoid dramatic rise of numbers of Infected and it is one of the main social aspect. As preventive measures we consider any pharmacological products which improve the immunity. Nowadays modern medicine offers many different products to customers, hence that provoke concurrence between medical products.

Assume on the pharmacological market there exist two products A and B which treat the same type of virus, but they differ from by some parameters (i.e. strength of product, price, etc.). Suppose that in total urban population one part of the population can use product A against the virus and other part use product B . As far as pharmacological products have different strength and price then we can say that

preferences of the individuals could be changed in long-run period. The main idea of the work is to research the influences of the concurrence between different pharmacologic products to the SIR model. The chain of individuals' choices is described by the evolutionary dynamics. According these assumptions extended SIR model is modified in following way, we define as (S_A) the group of people who use pharmacological product A and as (S_B) the group of people who use product type B . Individuals' choices influence to the epidemic process en masse.

As a main goal of this work we consider economic and epidemic aspects of impact the sickness rate in total urban population. The first aspect is minimization of treatment costs and costs of preventive measures. The second aspect is minimization the number of Infected in population to reduce epidemic effect. This complex problem is formulated in the class of optimal control problem, using instruments of the evolutionary games theory.

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Finding Nash and Stackelberg Equilibrium for Warehouse Inventory in Supply Chain Managment

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Keywords: Supply chain management, Nash equilibrium, Stackelberg equilibrium

Supply chain is known as one the most important subject on the economic science. In recent years after 2007, widespread researches have been done on supply chain management (SCM), proposed as an independent course and many books published on this subject. Unfortunately, most of the previous researches focused on the study of price effects on supply chain management and little works were done to study the effects of parameters such as inventory management, customer demand and waiting time and etc. In this work, we have studied the effect of inventory management on the supply chain as an important factor. Inventory management of manufactures is important to avoid two problems: loosing goods quality and profits. Inefficient inventory management causes storing surplus goods in storehouse and naturally lowering goods quality. On the other hand, it may decreases number of customers due to shortage in storehouse and subsequently reducing the profits.

Nash and Stackelberg inventory equilibriums have been found numerically and analytically. This paper describes the procedure to find the equilibriums with both methods by using different entrance distribution. Warehouse inventory equilibrium for uniform entrance distribution has been calculated with high level of accuracy both numerically and analytically approaches. We also extend the calculation of inventory equilibrium point to any desired entrance distribution by using numerical approach

A Smuggling Game Taking Account of Incomplete Information about a Smuggler

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Keywords: *Two-person zero-sum game, Incomplete information, Smuggling game.*

This report deals with a smuggling game, which is played during some days by Customs and a smuggler. Customs and the smuggler are allowed to take an action of patrol and smuggling, respectively, within the limited number of chances. Customs decides to patrol or not to patrol and the smuggler chooses one of smuggling or not smuggling every day. The capture of the smuggler gives Customs a reward and terminates the game. On the other hand, the smuggler gets a reward by the success of smuggling. The reward or the payoff of the game is assumed to be zero-sum. Almost all past researches modeled their games by the so-called complete information game and they assumed that each player knows the past strategies taken by his opponent or never knows them. Information about players is crucial to the results of the games. In this report, we deal with the smuggling game, taking account of incomplete information about the players. The smuggler knows past actions of Custom but Customs does not acquisition his opponent information, that is, the information acquisition is asymmetric between players. We propose a methodology to derive Bayesian equilibrium and evaluate the value of information.

The smuggling game originates from the so-called inspection game, which Dresher [2] begun. He dealt with a compliance problem for the treaty of arms reduction, where a violator wishes to violate the treaty for his benefit and an inspector wants to prevent the illegal behavior of the violator. Thomas and Nisgav [9] would be the first research on the smuggling game with multiple stages. They proposed a numerical algorithm of repeatedly solving a matrix game stage by stage. Baston and Bostock [1] gave a closed form of solution for the game similar to Thomas-Nisgav's model. After these researches, Garnaev [4],

Sakaguchi [8], Ferguson and Melolidakis [3] and Hohzaki [6,7] have been developing their smuggling game models from a variety of points of view.

In the past researches surveyed so far, they never thought of the asymmetrical information. The recognition of information plays an important role to the results of the game, as Harsanyi [5] pointed out by the concept of incomplete information. Generally speaking, players would acquisition information about their opponents in an asymmetric manner. Customs would be a public organization in many countries and the smuggler would be a secret society. Therefore, the behavior of Customs is comparatively open to outside but the smuggler's information tends to be kept in secret. Considering the practical situation, the information acquisition must be asymmetrical between players in the smuggling game.

In this report, we deal with the smuggling game with incomplete information and asymmetric information. First, we describe our smuggling problem in multiple stages or multiple days. Secondly, we elucidate the difference of observation between Customs and the smuggler, and formulate it by a Bayesian game. Thirdly, we develop the system of equations to solve the game and propose a numerical algorithm to derive Bayesian equilibrium point in a general case. At the same time, we derive analytical forms of equilibrium points in some special cases. Lastly, we analyze optimal strategies of players by some numerical examples and evaluate the value of information in the concrete.

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The Core and Nucleolus in a Model of Information Transferral

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Keywords: Core, Nucleolus, Shapley value, Information market game

Galdeano et al. (2010) introduced the so-called information market game involving n identical firms acquiring a new technology owned by an innovator. For this specific cooperative game, the nucleolus is determined through a characterization of the symmetrical part of the core. The non-emptiness of the (symmetrical) core is shown to be equivalent to one of each, super-additivity, zero-monotonicity, or monotonicity.

Consider the following problem. Besides n firms with identical characteristics, there exists an agent called the innovator, having relevant information for the firms. The innovator is not going to use the information for himself, but this information can be sold to the firms. Any firm that decides to acquire the new information is supposed to make use of the information. The n potential users of the information are the same before and after the innovator offers the new technology. The firms acquiring the information will be better than before obtaining it, while their utilities are computed under a conservator point of view, assuming that for any uninformed firm, the probability of making the right decision can be described by a *binomial probability distribution*, being $0 \leq p \leq 1$ the uniform probability of having success. The probability that k among n firms take the right decision is given by $\binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ and hence, the *expected aggregated utility*

of k firms having success is given by $k \cdot \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \cdot u_k$. Here $u_k \geq 0$ represents the utility if k firms make a right decision. Throughout the paper, the utility function is monotonic decreasing because when the number of firms taking the right decision

increases, each firm receives a lower utility level, i.e., $u_{k+1} \leq u_k$ for all $k \geq 1$ (not necessarily normalized in that $u_1 = 1$).

This information trading problem has been modeled by Galdeano et al. (2010) as a cooperative game $\langle N, v \rangle$ in characteristic function form, where the set of firms $N = \{1, 2, \dots, n+1\}$ consists of the innovator 1, having a new information, and the users $2, 3, \dots, n+1$, who could be willing to buy the new information. In case coalition S contains the innovator, then its worth $v(S)$ in the so-called information market game equals $(s-1) \cdot u_n$ because any member of S , different from the innovator, took the right decision rewarding the expected utility u_n since the $n-s$ uninformed firms outside S are assumed to take right decisions too.

Definition 1 *The $(n+1)$ -person information market game $\langle N, v \rangle$ in characteristic function form is given by, $v(\emptyset) = 0$, and on the one hand (cf. Galdeano et al., 2010),*

$$v(S) = (s-1) \cdot u_n \text{ for all } S \subseteq N \text{ with } 1 \in S \text{ and on the other,} \quad (1)$$

$$v(S) = f_n(s) = \sum_{j=1}^s j \cdot \binom{s}{j} \cdot p^j \cdot (1-p)^{s-j} \cdot u_{n-s+j} \text{ for all } S \subseteq N \setminus \{1\}, S \neq \emptyset \quad (2)$$

We claim the equivalence of three game properties (called super-additivity, zero-monotonicity, and monotonicity). The proof of their equivalence is based on the monotonic increasing average profit function for coalitions not containing the innovator. This significant property allows us to report an equivalence theorem.

Theorem 1 *For the $(n+1)$ -person information market game $\langle N, v \rangle$ of the form (1)-(2), the following four statements are equivalent.*

$$\text{Super-additivity} \Leftrightarrow \text{Zero-monotonicity} \Leftrightarrow \text{Monotonicity} \Leftrightarrow \frac{f_n(n)}{n} \leq u_n$$

Generally speaking, marginal contributions of players are well-known as upper bounds for pay-offs according to core allocations, that is $x_i \leq v(N) - v(N \setminus \{i\})$ for all $i \in N$ and all $\bar{x} \in \text{CORE}(N, v)$. Throughout the paper, given a pay-off vector $\bar{x} = (x_i)_{i \in N} \in \mathbb{R}^{n+1}$ and a coalition $S \subseteq N$, we denote $\bar{x}(S) = \sum_{i \in S} x_i$, where $\bar{x}(\emptyset) = 0$.

The core allocations are selected through *efficiency and group rationality*. The core, however, is a set-valued solution concept which fails to satisfy the symmetry property in

that users of the same type receive identical pay-offs according to core allocations. In order to determine the single-valued solution concept called nucleolus, being some symmetrical core allocation, our main goal is to investigate the symmetrical part of the core.

Definition 2 For any $(n+1)$ -person game $\langle N, v \rangle$,

$$(i) \text{CORE}(N, v) = \{\bar{x} \in \mathbb{R}^{n+1} \mid \bar{x}(N) = v(N) \text{ and } \bar{x}(S) \geq v(S) \text{ for all } S \subseteq N\} \quad (3)$$

(ii) The symmetrical core allocations require equal pay-offs to users, that is

$$\text{SymCORE}(N, v) = \{\bar{x} = (x_i)_{i \in N} \in \text{CORE}(N, v) \mid x_2 = x_3 = \dots = x_n = x_{n+1}\} \quad (4)$$

Theorem 2 For the $(n+1)$ -person information market game $\langle N, v \rangle$ of the form (1)-(2) with $0 \leq p < 1$, the following five statements are equivalent.

(i) The core is non-empty, $\text{CORE}(N, v) \neq \emptyset$

(ii) The symmetrical core is non-empty, $\text{SymCORE}(N, v) \neq \emptyset$

(iii) $b_1^v \geq 0$

(iv) $\frac{f_n(n)}{n} \leq u_n$

(v) $\{\text{Super-additivity, Zero-monotonicity, Monotonicity}\}$

Theorem 3 Suppose that the symmetrical core of the $(n+1)$ -person information market game is non-empty, that is $u_n \geq \frac{f_n(n)}{n}$. Let $1 \leq t \leq n$ be a maximizer in that

$$\frac{f_n(t) + u_n}{t+1} \geq \frac{f_n(s) + u_n}{s+1} \text{ for all } 1 \leq s \leq n. \quad (5)$$

Let $\bar{\alpha} = \frac{f_n(t) + u_n}{t+1}$ and $\bar{x}(\bar{\alpha}) = (n \cdot (u_n - \bar{\alpha}), \bar{\alpha}, \bar{\alpha}, \dots, \bar{\alpha}) \in \mathbb{R}^{n+1}$.

Then the nucleolus of the $(n+1)$ -person information market game equals the pay-off

vector $\bar{x}(\bar{\alpha})$ which belongs to the symmetrical core in that $\frac{f_n(n)}{n} \leq \bar{\alpha} \leq u_n$.

Theorem 4 Suppose that the symmetrical core of the $(n+1)$ -person information market game is empty, that is $\frac{f_n(n)}{n} > u_n$. Let $1 \leq t \leq n$ be a maximizer satisfying $\frac{f_n(t)}{t} > u_n$ in that

$$\frac{f_n(t) + n \cdot u_n}{t + n} \geq \frac{f_n(s) + n \cdot u_n}{s + n} \quad \text{for all } 1 \leq s \leq n. \quad (6)$$

$$\text{Let } \bar{\beta} = \frac{f_n(t) + n \cdot u_n}{t + n} \text{ and } \bar{x}(\bar{\beta}) = (n \cdot (u_n - \bar{\beta}), \bar{\beta}, \bar{\beta}, \dots, \bar{\beta}) \in \mathbb{R}^{n+1}.$$

Then the nucleolus of the $(n+1)$ -person information market game equals $\bar{x}(\bar{\beta})$.

Theorem 5 The Shapley value $Sh_1(N, v)$ of the innovator in the $(n+1)$ -person information market game $\langle N, v \rangle$ equals the difference between one half of the aggregate pay-off and the average worth of coalitions not containing the innovator, that is

$$Sh_1(N, v) = \frac{n \cdot u_n}{2} - \frac{1}{n+1} \sum_{s=0}^n f_n(s) \quad \text{and for all } i \in N, i \neq 1, \quad (7)$$

$$Sh_i(N, v) = \frac{1}{n} [v(N) - Sh_1(N, v)] = \frac{u_n}{2} + \frac{1}{n \cdot (n+1)} \cdot \sum_{s=0}^n f_n(s) \quad (8)$$

Urn Scheme for a Buying-Selling Problem

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Keywords: *Optimal stopping, Urn sampling, Ballot problem*

In this paper, the urn scheme for buying-selling problem is considered. There is an urn containing m balls of value -1 and p balls of value $+1$. The player is allowed to draw ball randomly, without replacement, one by one. The player observes the values of the balls and wants to make two stops. The player's goal is to stop with maximum probability at the first on the minimum of the sum of the balls' values and then on the maximum.

This urn scheme could be considered as the buying-selling problem. Here the value of the ball is change of the cost of an asset. The first stop means the buying of an asset and the second stop is the selling of an asset. The player wants to buy an asset by a low price and sell it by a high price.

The one-stop problem (max-problem) and its modifications was considered by Tamaki M. (2001). Others urn schemes were discussed by Shepp L. (1969) (net gain problem), Mazalov V.V., Tamaki M. (2007) (duration problem).

In this paper, the optimal stopping rule is derived and the asymptotic behavior of the player's payoff is investigated.

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Cooperative Behaviour in Social Networks

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Keywords: Social network, Game theory, Cooperative game

The paper deals with the two--stage cooperative game in a social network. The problem models profit sharing between shareholders.

Consider the social network built by n agents [1,2,3]. An opinion of the i -th agent in the time t defined by the $x_i^t \in [0,1]$, $i \in N = \{1, 2, \dots, n\}$, $t = 0, 1, 2, \dots$. An information impact of agent i to agent j defined by value $a_{ij} \geq 0$, $i, j \in N$. The impact matrix

$A = (a_{ij})_{N \times N}$ is stochastic by rows: $\sum_{j=1}^n a_{ij} = 1$. The agents have initial beliefs $x^0 = (x_i^0)_{i \in N}$

about the part of overall payoff that should be shared between the agents (e.g., what part of company's profit should be shared between shareholders). On the first stage the social network is reaching the consensus by interaction between agents. At each step the agent i changes her opinion in view of opinions of the other agents:

$$x_i^t = \sum_{j \in N} a_{ij} x_j^{t-1}, t = 1, 2, \dots; i \in N. \quad (1)$$

Interaction between the agents repeats until they have the common opinion. On the second stage the payoff is divided between players. We use cooperative game theory [4,5] in order to examine the possible coalitions of agents and to estimate the system's solutions.

We develop the basic model and consider the examples.

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Stochastic Game with Endogenous Transitions

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Keywords: *Stochastic games, Endogenous transitions, Limiting average rewards, Equilibria, Common pool resource dilemma.*

We introduce a stochastic game in which transition probabilities depend on the history of the play, i.e., the players' past action choices. To solve this new type of game under the limiting average reward criterion, we determine the set of jointly-convergent pure-strategy rewards which can be supported by equilibria involving threats.

We examine the following setting for motivational and expository purposes. Each period, two agents exploiting a fishery choose between catching with restraint or without. The fish stock is in either of two states, High or Low, and in the latter each action pair yields lower payoffs. Restraint is harmless to the fish stock, but it is a dominated strategy in each stage game. Absence of restraint damages the resource, i.e., the less restraint the agents show, the higher the probabilities that Low occurs at the next stage of the play. This state may even become 'absorbing', i.e., transitions to High become impossible.

N-Person Transportation Game with Different Cost Functions

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The computational algorithm for finding the shortest generalized trajectory of agents connecting some set of initial vertices with a given set of terminal vertices is proposed. The generalized trajectory may consist of paths having common edges. The transportation costs of different agents are different. The cost of passing through a common edge for each agent equals to the average cost. The algorithm is used to compute the values of characteristic function of corresponding cooperative game. This enables to compute effectively different optimal solutions of cooperative theory.



РОССИЙСКИЙ ЖУРНАЛ МЕНЕДЖМЕНТА

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Farsighted Stable Sets of Tariff Games

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Keywords: *Farsighted stable set, Tariff, International trade*

This article analyzes the tariff negotiation game between two countries when the countries are sufficiently farsighted. Primary papers in the literature, such as Johnson (1953-1954) envision a scenario in which countries choose an optimal tariff rate given that the other country does not change its tariff rate. Tower (1975) and Rodriguez (1974) have carried this analysis over to the game in which countries, instead of choosing the tariff rates, choose export or import quotas. Although not explicit in their formulation, their framework employs an equilibrium concept similar to that of Nash equilibrium. In these models, each country successively chooses a tariff rate or a quota level under the assumption that the other country stays put.

However, when each country chooses such an optimal level, it does not take into account the consequences of such actions that it triggers, including the possibility that the other country may retaliate in response. Recently, Nakanishi (1999), for the quota game, and Oladi (2005) and Nakanishi (2000), for the tariff game, have applied the theory of social situations of Greenberg (1990) to the export quota game and the tariff game respectively to capture this possibility in their model. However, the domination relation that their findings are based on does not take into account the situation in which players are not myopic. In this paper, we analyze the stable outcomes in tariff games when players can sufficiently take into account the consequences of their deviations and are only interested in the final outcomes as results of such deviations. To do so, we apply the farsighted stable set to tariff games.

There has been a growing literature of the application of farsighted stable set of Chwe (1994). The starting point of the argument for the farsighted stable set start with the argument by Harsanyi (1974) and Chwe (1994) that the classic stable set of von

Neumann and Morgenstern (1953) uses a domination relation that is myopic. Attempting to take into account sequences of deviations that may occur, Harsanyi (1974) and Chwe (1994) define a domination relation, called indirect domination, which is then used to define the farsighted stable set. This solution concept has been used in papers, including, to the authors' best knowledge, Suzuki and Muto (2005) and Kamijo and Muto (2010) in sending a message that farsightedness is the key element in reaching Pareto efficient outcomes. This message has to be taken cautiously since they allow coalitional deviations - that is, simultaneous deviation by multiple players. The juxtaposition of the results in Suzuki and Muto (2000), Masuda (2002), Nakanishi (2009), and Kawasaki and Muto (2009) reveal that there is not a direct relationship between the efficiency of the results in farsighted stable sets and the rules of the game ascertaining the allowance of coalitional deviations.

In light of the aforementioned papers in the literature, we analyze the farsighted stable sets of two different games of tariff games. In the first model, we allow for coalitional deviations - simultaneous deviations made by both countries. The first model corresponds to the rule of negotiating outlined in Oladi (2005) and the first model in Nakanishi (2000), both of which consider tariff retaliation games. In the second model, we disallow coalitional deviations. This restriction can be interpreted as an alternating negotiation game in which one player proposes one tariff, while in the next step, the other player can respond. This model is closely related to Nakanishi (1999), which also restricts deviations to those made by individual players in a quota retaliation game.

We show that in both games, the tariff choices by two countries that is Pareto efficient and strictly individually rational constitutes a singleton farsighted stable set. Moreover, we can show that no other farsighted stable sets exist in these two games. Thus, the rules of the game regarding coalitional deviations do not affect the outcome of the results, although the proof of the statement is far more involved in the second game. Unlike Nakanishi (2000), to achieve efficiency the only main addition to the original model in Oladi (2005) that is used is that countries are sufficiently farsighted, and in addition, outcomes that are not individually rational are not supported by a farsighted stable set.

One criticism to this approach is that it requires the players to be able to foresee events multiple steps ahead. However, as will be apparent in the proofs of the statements of this paper, we do not need to assume a substantial amount of farsightedness to

establish the results. All of the results hold when player can foresee at least four steps ahead.

Our main focus of this paper is on tariff games, but we can easily use the same logic employed in this paper to show a similar result for the export quota game.

In section 2, we introduce two models of the tariff game as mentioned above. In section 3, we review the literature on farsighted stable sets and provide key definitions and their properties. In sections 4 and 5, we present the results for the two models. We conclude in section 6



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DYNAMIC GAMES AND APPLICATIONS

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Assymmetric Information and Intermediation

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Keywords: *Assymmetric information, Intermediaion, Informed principal, Bileteral deviations*

In this paper we look at the game of asymmetric information where two agents make a collaboration decision not knowing the type of each other. Intermediary has supreme knowledge about types of agents and may provide his service to insure against the failure of the partner. We are looking for PBE that are stable to bilateral deviations. We find that intermediary may be welfare-improving in this setting. For all values of parameters there are equilibria where intermediary connects either only low type agents or only high type agents. When there is a market of intermediaries we can have three types of outcomes: the price of intermediation is very high, and the service is provided either only to low type agents or to everyone; the price is moderate and intermediary links everyone except for the pairs of low type agents; or the price is low and intermediary connects only high type agents.

Our preliminary analysis of two intermediaries case shows that the structure of equilibria there may be very rich and different from that of classical price competition.

JEL: D8, C7

Introduction

Intermediaries play an important role in promoting collaborations, especially when there are any kind of frictions in the market. For example, under direct trade with matching protocol trade may fail, moreover both buyers and sellers have incentives to misreport their willingness to pay or their costs. This may result in efficiency loss, or even in breakdown of trade. In general, intermediary may encourage link formation. For example in business networks presence of an intermediary can be efficient when search costs for business counterparts are sufficiently high or risks involved in a new link formation are large. Another interesting evidence on intermediation can be on mafia

networks, where introduction was the only possible way to form a new link. In "Ten Commandments" of Sicilian mafia it is written "No one can present himself directly to another of our friends. There must be a third person to do it." (Mastrobuoni, 2010).

Literature related to our model can be divided into several groups according to different aspects it is capturing that are addressed in our work.

In their paper on structural holes (2007) Goyal and Vega-Redondo explore motives for link formation when agents can either pay or extract rents from intermediation depending on their position in the network. They show that without capacity constraints the star network emerges, where the central player acts as an intermediary and enjoys significant rents from his position. On the contrary, with the presence of capacity constraints, cycle is the equilibrium network, with no one agent being an intermediary (as no one is essential for connecting any two others) and all getting the same payoffs.

A valuable group is literature on referrals. In "Social networks and labor-market outcomes: toward an economic analysis" J. Montgomery examines the role of employee referrals on the labor market with adverse selection. The model is two-period, and workers can be of two types - high and low. Workers that are employed in the first period recommend those linked to them. Ability of connected workers is exogenously correlated. Montgomery shows that in equilibrium companies will use possibility to hire only those second period workers that were introduced by high ability employees. In the second period, workers that were hired through some acquaintance receive wage below their expected productivity, therefore a firm gets a positive expected profit. As firms are competitive, the wage they are paying to the first period workers exceed their expected productivity because first period workers also have an optional value that can lead to a positive expected profit in second period. The author also investigates the impact of network structure on wage dispersion: increase in either network density or correlation between productivity of connected agents increases wage dispersion. However, in this model the reference decision of first period workers is non strategic - they always give references to any agents with whom they are connected.

Saloner in the work "Old boy networks as screening mechanism" considers dynamic model of references with reputation. There are more than one competing referees that have some signal about abilities of their candidates. Each referee wants more of his candidates to be hired, however he cares about their average quality. Thus, there is a trade-off between recommending more friends or recommending fewer of them

but of higher quality. Each referee uses a cut-off strategy. The model is multi-period, and the stationary equilibrium is found, where the firm has stationary beliefs about the average quality of those referred by each individual, and referees have stationary strategies about whom to recommend depending on the signal they get. As a result of the model, though referees act strategically, because of the competition resulting equilibrium is efficient (so the result is the same as if the firm itself got the signals that referees had).

Rubinstein and Wolinsky (1987) incorporate intermediaries in a bargaining and matching framework. In their work market consists of three types of agents - buyers, sellers and intermediaries. This model doesn't addresses advantages of intermediation as buyers and sellers meet a middleman by some exogenous process.

Large strand of literature considers the role of intermediary in trade. In his paper Gehrig (1993) looks at the intermediary in the market with costly search. Buyers and sellers choose between direct trade or trade through an intermediary who purchases and sells products. Intermediary offers the service of immediacy by posting directly bid and ask prices and allows agents to avoid the costly search. The author finds that traders with low gains from trade are not willing to pay intermediary costs and will go for a direct trade. Monopolist intermediary charges positive spread, while in case of competition the classical Bertrand result applies.

Stahl (1988) and Yannelle (1989) in their works look at two-sided price competition. They find that non-Walrasian equilibria with positive bid-ask spreads may emerge, even when intermediation technology is costless because intermediaries offer attractive bid price, and obtain monopoly position towards buyers. Moreover, the existence of equilibrium may be problematic.

Garella (1989) looks at trade with asymmetric information, and finds that intermediation may complete the market system when asymmetric information causes failure without one. This result is obtained under the hypothesis that the intermediary randomizes the price offers to the seller.

Every time when there are intermediaries they are competing against decentralized exchange. In our model we look at the market with asymmetric information, when agents can be either engaged in activity directly bearing the risk of collaboration themselves, or can use an intermediary as a buffer if he offers them such a choice. In our model from the point of view of economy as a whole connection through an intermediary is costless, while direct connection is costly. Therefore, in the economy intermediary not only has informational advantages, but also has efficiency gains.

However, from the point of view of two agents, direct connection might be more expensive than connection through an intermediary. We assume that intermediary already is connected to the agents and therefore he does not need to spend any extra resources. On the contrary, if agents would like to be connected to each other they would need to spend some effort on this. For example, suppose that a football player is looking for a new club to join, and a club is looking for a player to fill a position. Both a player and a club might find each other without any outside help, however they would need to incur costs, for example scouts have to fly to Brasil to look at local players. Moreover, both sides in such kind of matching have some uncertainty if the outcome of collaboration would be successful. Other way, they could be matched by an agent or even an agency. For such an agency cost of making a match is significantly lower. Moreover, it knows ability of the player, and in general has a good idea about potential of different teams.

An example that we have in mind through the paper is the case of so-called talent agents, or more precise - a booking agencies. These agencies mostly operate in musical industry and they differ in several aspects from talent agents. Booking agencies are those who book shows for artists. Essentially, they act as a middleman between band manager and a promoter. They have number of musicians whom they work with - usually, the whole list can be seen on their web-site. For example The Agency Group Ltd. has a combined roster of over 1000 artists and has local offices in five cities which allows to cover all international destinations. They work with such musicians as Zucchero, Apocaliptica, The Wailers, Muse.

A Non-antagonistic Differential Two-Person Game with Dynamic Disturbances

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Keywords: *Non-antagonistic differential game, Dynamic disturbances, Guaranteed payoffs, Nash equilibrium*

In considered game two players act in the class of strategies with memory, while disturbance has realizations in the class of Lebesgue measurable functions. The guaranteed payoffs for players

are defined. Nash equilibrium solution of such the game is determined. It is unprofitable for any player to deviate from the solution unilaterally. The unprofitableness is understood in sense of non-increase of guaranteed payoff for the deviating player.

Dynamics of a two-person non-antagonistic differential game (NADG) is described by the equation

$$\dot{x} = f(t, x, u, v, w) \quad u \in P, v \in Q, w \in S, \quad x[t_0] = x_0 \quad (1)$$

where $x \in R^n$ is a phase vector, u and v are controls of the first player (P1) and of the second player (P2), and w is a disturbance. The sets P , Q and S are compacta in R^P , R^Q and R^S , respectively. The function $f: G \times P \times Q \times S \rightarrow R^n$ is continuous in all variables, satisfies the Lipschitz condition in x and a condition of extendability on the given interval $[t_0, \vartheta]$. Here G is compact set in $R^1 \times R^n$ whose projection on the time axis is equal to the given interval $[t_0, \vartheta]$. Assume that all the trajectories of system (1), beginning at an arbitrary position $(t_*, x_*) \in G$ remain within G for all $t \in (t_*, \vartheta]$.

Cost functionals of P1 and of P2 are given by

$$I_i = \sigma_i(x[\mathcal{G}]), \quad i=1,2$$

where σ_i are continuous functions; \mathcal{G} is the fixed final time of the game.

Assume that both players know the equation (1) and the constraints on values of players' controls and of disturbance in (1). At the same time players have no other information about the realization of disturbance $w[t], t_0 \leq t \leq \mathcal{G}$. Assume also, that both players know the whole of prehistory of the phase vector $x[\cdot, t_0, t] = \{x[\tau], t_0 \leq \tau \leq t\}$ at the current moment of time t . Then, according to [1,2], both players act in the class of the strategies with memory. Namely, a strategy of P1 is identified with a pair $U \div \{u(t, x[\cdot, t_0, t], \varepsilon), \beta_1(\varepsilon)\}$, where $u(\cdot, \cdot, \cdot)$ is a functional defined by $t \in [t_0, \mathcal{G}]$ for all continuous functions $x[\cdot, t_0, t]$, all values of precision parameter $\varepsilon > 0$ and having values in the set P . For fixed ε the value $\beta_1(\varepsilon)$ is the upper bound for the step δ of a subdivision of the interval $[t_0, \mathcal{G}]$ which P1 uses for forming approximated motions. A strategy of P2 $V \div \{v(t, x[\cdot, t_0, t], \varepsilon), \beta_2(\varepsilon)\}$ is defined analogously.

Suppose that any Lebesgue measurable function $w[t], t_0 \leq t \leq \mathcal{G}$, satisfying at almost all t the condition $w[t] \in S$, can be chosen as a realization of disturbance. Such class of admissible realizations of disturbances will be denoted by Ξ .

An approximated motion (Euler broken line) $x[t, t_0, x_0, U, \varepsilon_1, \Delta_1, V, \varepsilon_2, \Delta_2, w[\cdot]]$ generated by the pair of strategies (U, V) and corresponding to the realization of disturbance $w[\cdot] \in \Xi$ is introduced for fixed values of players' precision parameters ε_1 and ε_2 and for fixed subdivisions $\Delta_1 = \{t_i^{(1)}\}$ and $\Delta_2 = \{t_j^{(2)}\}$ of the interval $[t_0, \mathcal{G}]$ chosen by P1 and P2 under the conditions $\delta(\Delta_i) \leq \beta_i(\varepsilon_i)$. A limit motion generated by the pair of strategies (U, V) from the initial position (t_0, x_0) is a continuous function $x[t] = x[t, t_0, x_0, U, V]$ for which there exists a sequence of approximated motions $\{x[t, t_0^k, x_0^k, U, \varepsilon_1^k, \Delta_1^k, V, \varepsilon_2^k, \Delta_2^k, w^k[\cdot]]\}$ uniformly converging to $x[t]$ on $[t_0, \mathcal{G}]$ as $k \rightarrow \infty$, $\varepsilon_i^k \rightarrow 0$, $t_0^k \rightarrow t_0$, $x_0^k \rightarrow x_0$, $\delta(\Delta_i^k) \leq \beta_i(\varepsilon_i^k)$, $w^k[\cdot] \in \Xi$. The set of these

motions is denoted by $X(t_0, x_0, U, V)$. By setting $\varepsilon_1 = \varepsilon_2$ only coordinated approximated motions and limit motions will be considered.

For given pair of strategies (U, V) and initial position (t_*, x_*) the guaranteed payoff of player i

is defined as follows

$$\rho_i(t_*, x_*, U, V) = \min_{x[\cdot] \in X(t_*, x_*, U, V)} \sigma_i(x[\mathcal{G}]).$$

A pair of strategies (U^N, V^N) forms Nash equilibrium solution of the game, if it is unprofitable for any player to deviate from the solution unilaterally. The unprofitableness is understood in sense of non-increase of guaranteed payoff for the deviating player.

Two auxiliary antagonistic positional differential games Γ_1 and Γ_2 are introduced, in each of which one player maximizes his guaranteed payoff, and other player together with a disturbance counteracts it.

Further problems of a finding of such pairs of players' strategies which provide to players payoffs, not smaller guaranteed in auxiliary games Γ_i are formulated. The set of the found pairs of strategies gives a basis for Nash equilibrium solutions in initial game.

Note that if for antagonistic differential games expansion of a class of pure strategies to a class of strategies with memory does not influence the result of the game, then for non-antagonistic differential games such expansion of classes of strategies already influences essentially, in general, this result.

The example of a finding of Nash equilibrium solutions in a game on the plane in the presence of a dynamic disturbances is considered.

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Quantitative Modeling of Strategically Stable Technological Alliance

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Consider a cooperative differential game $\Gamma(x_0, T - t_0)$ on a given finite time interval $[t_0, T]$, which involves 3 companies. Technological dynamics of the firm $i \in N = \{1, 2, 3\}$ is determined by the differential equation:

$$\begin{aligned} \dot{x}_i(s) &= \alpha_i [u_i(s)x_i(s)]^{1/2} - \delta x_i(s), s \in [t_0, T] \\ x_i(t_0) &= x_i^0, i \in N \end{aligned},$$

where $x_i(s) \in X_i \subset R_+$ is the technological level (state) of firm i ;
 $x(s) = (x_1(s), x_2(s), x_3(s))$ is the state vector at moment s ;
 $x(t_0) = (x_1(t_0), x_2(t_0), x_3(t_0)) = (x_1^0, x_2^0, x_3^0) = x^0$ is vector of firms' initial states;
 $u_i(s) \in U_i \in comp R_+$ is a control of firm i .

On the right-hand side of equations imposed conditions which guarantee the existence, uniqueness, and extendibility of solutions for any piecewise continuous controls $u_i(s), s \in [t_0, T]$.

Each firm seeks to maximize its profit, which is defined by an integral functional:

$$\int_{t_0}^T \left[P_i[x_i(s)]^{1/2} - c_i u_i(s) \right] \exp[-r(s-t_0)] ds + \exp[-r(T-t_0)] q_i [x_i(T)]^{1/2}$$

Firms form cooperation or technological alliance for maximizing of total profit. Participants receive additional cooperation opportunities in the development of technology through technology synergies in cooperation. Therefore, firms technological dynamics changes and takes the form:

$$\dot{x}_i(s) = \alpha_i [u_i(s)x_i(s)]^{1/2} + b_j^{[j,i]} [x_j(s)x_i(s)]^{1/2} + b_k^{[k,i]} [x_k(s)x_i(s)]^{1/2} - \delta x_i(s), s \in [t_0, T]$$

$$x_i(t_0) = x_i^0, \quad i, j, k \in N, i \neq j \neq k$$

where $b_j^{[j,i]} [x_j(s)x_i(s)]^{1/2}$ represents the technology transfer effect from j to firm i .

The profit of technological alliance is defined as the total profit of all participants, i.e.:

$$\int_{t_0}^T \sum_{i=1}^3 \left[P_i [x_i(s)]^{1/2} - c_i u_i(s) \right] \exp[-r(s-t_0)] ds + \sum_{i=1}^3 \exp[-r(T-t_0)] q_i [x_i(T)]^{1/2}$$

The problem of maximization of coalition payoff in this model was considered by Petrosyan and Yeung with method of dynamic programming and continuously differentiable Bellman function $W^{(t_0)K}(t, x_K^t)$, which determine max payoff of coalition $K \subseteq N$ in the sub game $\Gamma(x(t), T-t)$ of game $\Gamma(x_0, T-t_0)$. We also assume that the Bellman function is a coalition superadditivity, i.e.

$$W^{(t)K}(t, x_K^t) \geq W^{(t)L}(t, x_L^t) + W^{(t)K \setminus L}(t, x_{K \setminus L}^t), L \subset K$$

where $K \setminus L$ is relative addition L in K .

Profit is split between firms according to the chosen principle of optimality with the Profit Distribution Procedure (PDP). In this game, as the principle of optimality chosen the Shapley value.

At initial moment t_0 and initial state x^0 firms agree, that firm's i share of cooperative payoff will be equal the accordant component of the Shapley Value:

$$v^{(t_0)i}(t_0, x_N^0) = \frac{1}{6} W^{(t_0)i}(t_0, x_i^0) + \frac{1}{3} \left(W^{(t_0)\{i,j\}}(t_0, x_{\{i,j\}}^0) - W^{(t_0)j}(t_0, x_j^0) \right) +$$

$$\frac{1}{3} \left(W^{(t_0)\{i,k\}}(t_0, x_{\{i,k\}}^0) - W^{(t_0)k}(t_0, x_k^0) \right) + \frac{1}{6} \left(W^{(t_0)N}(t_0, x_N^0) - W^{(t_0)\{j,k\}}(t_0, x_{\{j,k\}}^0) \right)$$

$$i, j, k \in N \quad \tau \in [t_0, T]$$

The Shapley value has to be maintained throughout the game $[t_0, T]$, therefore at each moment $\tau \in [t_0, T]$ the next condition has to be maintained:

$$v^{(\tau)i}(\tau, x_N^{*\tau}) = \frac{1}{6} W^{(\tau)i}(\tau, x_i^{*\tau}) + \frac{1}{3} \left(W^{(\tau)\{i,j\}}(\tau, x_{\{i,j\}}^{*\tau}) - W^{(\tau)j}(\tau, x_j^{*\tau}) \right) + \\ \frac{1}{3} \left(W^{(\tau)\{i,k\}}(\tau, x_{\{i,k\}}^{*\tau}) - W^{(\tau)k}(\tau, x_k^{*\tau}) \right) + \frac{1}{6} \left(W^{(\tau)N}(\tau, x_N^{*\tau}) - W^{(\tau)\{j,k\}}(\tau, x_{\{j,k\}}^{*\tau}) \right) \\ i, j, k \in N$$

For realization of the dynamic Shapley value the joint payoff is distributed at each moment of game. Components of the Shapley value represents in following form:

$$v^{(t_0)i}(t_0, x_N^0) = \int_{t_0}^T B_i(s) \exp[-r(s-t_0)] ds + \exp[-r(T-t_0)] q_i(x_i^*(T))^{1/2}$$

where $B_i(s)$ is payment, received by firm i at moment s .

At each moment $t \in [t_0, T]$ the next condition has to be maintained:

$$v^{(t_0)i}(t, x_N^{*t}) = \int_t^T B_i(s) \exp[-r(s-t)] ds + \exp[-r(T-t)] q_i^i(x_i^*(T))^{1/2}$$

This cooperative solution will be dynamically stable. In addition, it defines a Nash equilibrium in game $\Gamma(x_0, T-t_0)$. Thus, the decision will also be strategically stable. There are quantitative examples showing that this resistance.

Bargaining about Meeting Time

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Keywords: *Bargaining, mMeeting time, Subgame perfect equilibrium, Utilities*

Suppose that players I_1, \dots, I_n like to choose a time x for the meeting, $x \in [0, 1]$. The preference of each player I_i is described by a continuous and strictly quasi-concave utility function $u_i : [0, 1] \rightarrow R_+$ with peak c_i .

The players make the proposals in discrete time $t = 0, 1, \dots$ with an infinite horizon. At $t = 0$ player I_1 proposes a time $x \in [0, 1]$ and the rest of players either accept or reject it. If x is accepted by all players then x is chosen and the game ends. Otherwise, the game moves to period $t = 1$ where I_2 proposes some offer. And so on, in order $I_1 \rightarrow I_2 \rightarrow \dots \rightarrow I_n \rightarrow I_1 \rightarrow \dots$. The process continues until an offer is accepted by all players. At time t the utilities of all players are discounted with factor $\delta \in (0, 1)$.

We find stationary subgame perfect equilibrium in case of piecewise-linear utilities with equal peaks at the points $c_1 = 1$, $c_2 = 0$, $0 \leq c_k \leq 1$ for $k = 3, \dots, n$

$$u_k(x) = \begin{cases} \frac{x}{c_k}, & \text{if } 0 \leq x \leq c_k, \\ \frac{1-x}{1-c_k}, & \text{if } c_k \leq x \leq 1. \end{cases}$$

Denote $0 = c^n \leq c^{n-1} \leq \dots \leq c^2 \leq c^1 = 1$ peaks in decreasing order. In asymptotic case as $\delta \rightarrow 1$ the optimal strategies are

$$x^* = \begin{cases} \frac{1}{n} & \text{if } c^2 < \frac{1}{n}, \\ \dots \\ \frac{k}{n} & \text{if } c^{k+1} < \frac{k}{n} < c^k, \\ c^{k+1} & \text{if } \frac{k}{n} \leq c^{k+1} \leq \frac{k+1}{n}, \\ \dots \\ \frac{n-1}{n} & \text{if } \frac{n-1}{n} < c^{n-1}. \end{cases}$$



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Game-Theoretic Models of Collaboration among Economic Agents

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Keywords: *Collaboration, Nash equilibrium, Value at Risk (VaR), Quantile*

Models explaining the mechanisms of situations, in which it is appropriate for economic agents to collaborate and act together despite of having independent goals have become rather interesting in both theoretical and application way. An interaction of economic agents, where each of them takes actions that bring direct benefit not only to him but to other agents can serve as the simplest example. An expectation of a beneficial counter action is an incentive for each agent to behave in this way. The most important difference between this behavior model from the «classical» models of rational economic agent's utility optimization is that here the utility of each agent depends directly on decisions made by others, whom he can indirectly influence.

Obviously, collaboration (in the context in which we agreed to consider it) and related issues may arise, for example, between the parties of public and private partnership, alongside with major investment projects or different schemes of financing from various levels of budget sources. Moreover, such models can also be useful in situations that go beyond «pure» economics. For instance, they can be applied to studies of intergovernmental negotiation processes aimed at achievement of agreements, which will complexly take both economic and political interests of the parties into account.

We will consider a simplified situation in order to explain the fundamental ideas of the proposed model. It describes interaction between two parties (agents, participants, players) $i = \{1, 2\}$, who are deciding upon the value of their own contribution to some

common project. This contribution (degree, depth of it) is quantitatively characterized by some arbitrary value x_i from 0 to 1: where «0» stands for lack of affirmative action in the project (non-collaboration, extremely selfish behavior, etc.), and «1» reflects the highest possible level of affirmative action (the maximum propensity to collaborate, ultimately constructive behavior).

Taking into consideration previously set objectives when defining the utility functions of players, we assume that the input (costs) performed by the agents reduce utility they can get, the utility can increase due to the inputs of his opponents. Linear relations are acceptable in model, as they will adequately reflect its fundamental properties. So we will define the utility function of the players, as

$$u_1(x_1, x_2) = -a_1x_1 + b_1x_2 \text{ and } u_2(x_1, x_2) = b_2x_1 - a_2x_2.$$

Accordingly, a_i is a value (score, a measure of regret) of a resource unit spent (invested in the project) by the player i and b_i is utility (effect, measure of satisfaction) for the i -th player, which he gets from a unit invested in the project by another party.

Having presented such a situation as «classical» finite non-cooperative two-person game, we face the fact that it has an obvious Nash equilibrium in pure strategies $x_1^* = x_2^* = 0$.

Indeed our productivity functions are arranged in such a way that the best response of the first player to any player's strategy will be to reduce his share of participation to zero. Thus, if we follow the concept of Nash equilibrium, we arrive at a pessimistic conclusion. At first we will concentrate on the approaches associated with transformation the original game to its mixed extension. Due to the fact that this scenario is based on a continuous set of pure strategies of the players, it seems obvious to set their mixed strategies as probability distributions with densities $p_1(x_1)$ and $p_2(x_2)$ on the interval $[0,1]$.

According to this, a particular choice of strategies by players in a particular round of the game can be interpreted as an implementation of independent random variables \tilde{x}_1 and \tilde{x}_2 .

This idea of mixed strategies of the players is a natural generalization of the «traditional» definition of mixed strategies in matrix and bimatrix games, which can be defined as likelihood (p_1, \dots, p_k, \dots) with which every player will implement one or

another pure strategy. Following this logic, we would have had to sample the interval $[0,1]$ in order to bring in traditional «discontinuous» mixed strategies. This method, however, seems to be not enough justified and reasonable in terms of reflecting the economic realities.

When mixed strategies are defined in the form of continuous distributions, a player's strategic choice is generally reduced to the choice of parameters of these distributions. Due to the fact that the number of parameters in different probability distributions classes is different, we come to a conclusion that definition of the players' strategies within the stated model will vary according to the type of distribution $p_1(x)$ or $p_2(x)$ we've chosen. Actually the value of strategies chosen by the participants in each act of the game can be viewed as a realization of independent random variables \tilde{x}_1, \tilde{x}_2 , whose densities are known; and utilities $u_1(\tilde{x}_1, \tilde{x}_2), u_2(\tilde{x}_1, \tilde{x}_2)$ are determined as functions of random variables, the characteristics of which, generally speaking, can be determined with the help of $p_1(x), p_2(x)$.

We should note that specification of participants' strategic choices in the form of continuous probability distributions can be justified by the theory of evolutionary games. Namely, we can assume that we have a community consisting of groups (populations). Different populations of players have different tendencies to collaborate (collaborative behavior). These tendencies are realizations of random variables \tilde{x}_i with densities $p_i(x)$. When members of different populations confront in some acts of the game, their success (or lack of success) can be expressed in terms of utility \tilde{u}_i . After that evolution of stochastic characteristics of propensity to cooperate takes place and these indicators reach some «benchmark» stable states, based on the experience accumulated by populations. Of course, if the strategies participants are determined with continuous probability distributions, we can only compare them correctly if function $p_i(x)$ is restricted by some single parametric class \mathbf{P}_i . In this case, parameters of density functions $p_i(x)$ become «natural» characteristics of strategies. Accordingly, the set of possible situations in a game is defined by the set of all possible combinations $p_i(x)$ of all players.

In terms of the classical Nash approach , the equilibrium (solution) in the described model will be characterized by such joint choice of probability distributions $(p_1^*(x), p_2^*(x))$ from which each of the participants in the game would not be advantageous to deviate from separately, i.e.:

$$\mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2^*(x)\} \geq \mathbf{E}\{u_1(\tilde{x}_1, \tilde{x}_2) \parallel p_1(x), p_2^*(x)\},$$

$$\mathbf{E}\{u_2(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2^*(x)\} \geq \mathbf{E}\{u_2(\tilde{x}_1, \tilde{x}_2) \parallel p_1^*(x), p_2(x)\},$$

for every $p_1(x) \in \mathbf{P}_1$, $p_2(x) \in \mathbf{P}_2$, where $\mathbf{E}\{u_i(\tilde{x}_1, \tilde{x}_2) \parallel p_1(x), p_2(x)\}$ is the expected value of $u_i(\tilde{x}_1, \tilde{x}_2)$, calculated on the assumption that distribution \tilde{x}_1 is determined by the density function $p_1(x)$, and distribution \tilde{x}_2 by the density function $p_2(x)$.

Since a randomized model is being described, we cannot deny admissibility and validity of alternative approaches, which determine the equilibrium conditions with respect to other criteria. Particularly, they may be:

- minimization of variances of players' utilities (perhaps with additional restrictions on the lower levels, below which the utility expectation value cannot go);
- minimization of α -quintile values of players' utility function distributions, that is, such values, below which the value of the utility will not fall with a probability $1-\alpha$.

Socially Concerned Firms and Endogenous Choice of Strategic Incentives

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Keywords: *Socially concerned firms; Corporate social responsibility; Strategic incentives; Mixed oligopoly*

In the literature several approaches have been proposed to ensure the sustainability over time of an agreement reached at the starting date of a differential game. One of the approaches appropriate for two-player differential games is to support the cooperative solution by incentive strategies, [2], [3]. Incentive strategies are functions of the possible deviation of the other player and recommend to each player to implement his part of the agreement whenever the other player is doing so. The equilibrium incentive approach allows embodying the cooperative solution with an equilibrium property. Therefore, by definition each player will find individually rational to stick to his part in the coordinated solution. One important property that should be checked is the credibility of these incentive strategies, [3], [4]. These strategies are credible if each player will implement his incentive strategy and not the coordinated solution if he observes that the other one has deviated from the agreement. Recently some papers, [5], [6], have provided conditions to check for the credibility of incentive strategies for the class of linear-state and linear-quadratic differential games. To preserve the special structures of the games the analyses have been restricted to linear incentive strategies that are not always credible.

Corporate Social Responsibility (henceforth CSR) has become mainstream and the majority of managers believes that CSR creates shareholder value, results in a competitive advantage and in cost savings (e.g. Fortune 2003). However, theoretical research on strategy and governance issues has largely neglected the topic of CSR until recently (see Kopel 2011 for a literature review). Since socially concerned firms are active in the same markets as profit-maximizing firms, it is of considerable interest to

ask which goals socially concerned firms might pursue and how their presence affects the firms' performance and welfare (Goering 2010, 2008a, b, Becchetti and Hybrechts 2008, Casadesus-Masanell and Ghemawat 2006, but also Marwell and McNerney 2005, Schiff and Weisbrod 1991, and Lien 2002). Likewise, it also seems important to consider the organizational governance of these socially responsible firms, i.e. how their organizational structure and incentive systems differ from those of firms with other objectives (e.g. Berrone and Gomez-Mejia 2009; Mahoney and Thorne 2005, 2006, Frye et al. 2006) and which differences in management's behavior are induced (Berger et al. 2007, Du Bois et al. 2004). Furthermore, the interaction between the firms' governance and product market competition is interesting and worthwhile to study.

In our paper we consider a dynamic game which addresses this management problem. A profit-maximizing firm competes against a socially concerned firm in a linear homogenous-product duopoly. In contrast to the profit-maximizing firm, the socially responsible firm is assumed to maximize an objective function which takes its profit plus a share of consumer surplus into account (see also Lambertini and Tampieri 2010, Goering 2007, 2008a,b, Lien 2002). To include organizational governance aspects in the model, we assume that both firms have the option to hire a manager, who is taking over the responsibility to determine the production quantity on behalf of the firms' owners. If firms hire a manager, they write incentive contracts for their managers to provide strategic incentives. Within this model, we try to answer the following main research questions: (i) Will both firms hire managers and delegate the production decision? (ii) Does it pay off for a firm to be socially concerned, i.e. can it yield a competitive advantage to pursue goals different from profit and compensate the manager for it? (iii) What is the impact of an increasing concern for consumer welfare on prices, quantities, industry profits and welfare?

Our multi-stage game yields the following insights. In the subgame-perfect equilibrium, both firms' dominant strategy is to hire a manager and delegate the production decision. If the profit-maximizing firm and the socially responsible firm have identical unit production costs, then the socially responsible firm has a higher market share than the profit-maximizing firm and obtains a higher profit. A comparative statics analysis shows a monotonic increase of the value of the objective function if the share of consumer surplus is increased. On the other hand, we find a non-monotonic relationship between the equilibrium profit of the socially concerned firm and the share of consumer surplus the firm includes in its objective function. The firm's profit first increases if this

share is increased, but then decreases. In the light of the revived discussion about stakeholder or shareholder view, the finding that a firm's profit can simultaneously achieve both goals, that is increase the value for its stakeholders and at the same time increase its profit, even if it competes head-on against a profit-maximizing rival, is interesting. The reason here is that in a situation of strategic interaction accounting partially for a stakeholder group (here the consumers) can serve as a commitment device and can result in an increase in the socially concerned firm's competitive advantage. In this sense, ... investing in stakeholder management may be complementary to shareholder value creation and may indeed provide a basis for competitive advantage ... (Hillman and Keim 2001, p. 135). This result shows that non-profit maximizing firms competing against profit-maximizing rivals in an imperfect market can indeed have an advantage (see Kelsey and Milne 2008). It is important to notice, however, that such a social concern is rewarded only up to a point, that is "it pays to be good, but not too good" (Mintzberg 1983). In other words, it pays off to pay attention to stakeholders, but not too much. If the firm already puts a high weight on consumer surplus, increasing the weight even further destroys shareholder value. Furthermore, we also find that for an increase in this share, industry output and total welfare increases. If the profit-maximizing firm and the socially responsible firm have different unit production costs, then the comparison obviously depends on the cost differential. However, if the cost difference is not too large, then the insights of the symmetric cost case carry over to the more general situation with asymmetric costs.

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Dynamic Stability of Strategic Alliances in Automobile Industry

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Keywords: *Dynamic stability, strategic alliances, automobile industry, game theory.*

Firms create alliances to achieve additional value. Or, firms cooperate in order to produce a product together, to gain scale advantages, to enter new market, to learn from each other or to establish technological cooperation.

Interaction is important in both alliance formation and partnership dynamics. Alliances are the result of an interaction among several firms. Both forming and managing an alliance is the result of a collective effort. Furthermore, strategic alliances are intentionally long-term. Thus, it is expected that the game theoretical perspective implemented in the study will be a useful instrument in providing a dynamic approach to alliances.

The main subject of the study is alliance dynamics. One of the indicators of a good alliance performance is alliance stability, when the alliance evolves to the satisfaction of all the partners. Cooperation makes the partners dependent on each other; each firm needs its partners to achieve a good performance. This dependence is complicated by the uncertainty the partners face with regard to the behavior of their partners. When participating in strategic alliance, each firm wants to reach the potential value of the alliance by committing to the alliance. But, the benefits of the alliance are distributed among its participants. Thus, participants can have an incentive to behave opportunistically. In other words, firms should choose between commitment and opportunism. Hence, game theory is well-suited to study how alliances are formed and how partners interact in a dynamic perspective.

The research considers strategic alliances in automobile industry (Renault-Nissan alliance, DaimlerChrysler Company, Peugeot Citroen alliance). The amount of

strategic alliances in automobile industry has grown in last years. The industry is characterized by complex production, high performance requirements, high R&D costs, rapidly changing technology and a very competitive marketplace. All these factors lead to a natural incentive of the firms to form strategical partnerships to gain additional value.

In the study historical financial data of the alliances is used as indicator of alliances performance. The aim of the research is to investigate whether the strategic alliances are dynamically stable or not, what optimal principles of cooperative behavior the partners within alliance follow, what factors can cause a long-term stability of the alliance, and what factors can cause the failure of the alliance. For instance, it has been shown that the Renault-Nissan alliance is dynamically stable, and that the partnership of the alliance is guided by Pareto optimality principle.

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Strategic Pricing of Complementary Products in a Marketing Channel

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Keywords: *Pricing Strategies, Complementary products, Marketing channels, Game Theory.*

The aim of this paper is to characterize equilibrium pricing strategies in a marketing channel formed of a manufacturer and two retailers. The particularity of our setting lies in the fact that the manufacturer offers a product that can be used only by consumers who own a complementary product sold by the two retailers. To fix ideas, the manufacturer's product is an ebook, whereas the complementary products are Kindle (by Amazon) and Tablet (by Sony). In particular, we discuss under which conditions it is in the best interest of the retailer selling both base and complementary products to practice a loss-leader pricing strategy. Also, we investigate the impact of presence of the complementary product on vertical inefficiencies and profits of the different players.

On A Nonrenewable Resource Extraction Game Played by Asymmetric Firms

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Keywords: Differential game, Exhaustible resources, Random terminal time, Hamilton-Jacobi-Bellman equation.

A class of models of differential games where 2 firms are engaged in a competition of extraction of a nonrenewable resource is analyzed. In particular, a framework where the terminal instants of extraction are random variables having different cumulative distribution functions is considered. The first firm which stops extracting is the loser, whereas the remaining firm gets a terminal reward and keeps extracting on its own until the exhaustion of the resource.

Two firms involved in a noncooperative differential game of resource extraction with the following setup are considered:

- given the different characteristics of the 2 firms, each one of them has a distinct terminal time of extraction of the same resource;
- as soon as the first one finishes, it quits the game and there remains just one firm left, which keeps extracting until its terminal time;
- the payoff of the game is composed of two components: the integral payoff achieved while playing, and the final reward, assigned to the player which stays alive after the retirement of its rival;
- the control variables of the players are their respective extraction rates $u_1(t), u_2(t) \in \mathbb{R}_+$;
- the unique state variable of the game is the stock of resource $x(t) \in \mathbb{R}_+$, whose evolutionary dynamics is expressed by the following differential equation:

$$\begin{cases} \dot{x}(t) = \phi(t, x, u_1, u_2) \\ x(0) = x_0 > 0 \end{cases}, \quad (1)$$

where the transition function $\phi(\cdot) \in C^2(\mathbb{R}_+^4)$ is negatively affected by the firms' extraction efforts:

$$\frac{\partial \phi}{\partial u_i} \leq 0, \text{ for } i = 1, 2;$$

- denote by $h_i(t, x, u_1, u_2) \in C^2(\mathbb{R}_+^4)$ the utility function of the i -th firm. No intertemporal discount factor appears in the functional objectives of the problem, because the discount structure is built on the characteristics of the random terminal instants.

Let T_1 and T_2 be the random variables denoting the respective terminal instants of the extracting firms, and assume that their c.d.f. $F_1(\cdot)$, $F_2(\cdot)$ and their p.d.f. $f_1(\cdot)$ and $f_2(\cdot)$ are known.

We impose an asymmetry condition concerning the longevity of players: calling $\omega_i > 0$ the upper bound of T_i , it is not restrictive to posit $\omega_1 > \omega_2$. Hence, the two p.d.f. naturally differ:

$$\begin{aligned} F_1(t) &< 1 \quad \forall t < \omega_1, \quad F_1(\omega_1) = 1; \\ F_2(t) &< 1 \quad \forall t < \omega_2, \quad F_2(t) = 1 \quad \forall t \in [\omega_2, \omega_1]. \end{aligned}$$

At time $T = \min\{T_1, T_2\}$, if player i is the only one remaining in the extraction game, she receives the terminal payoff $\Phi_i(x(T))$, subsequently, since she keeps playing on her own, the game collapses to an optimal control problem.

If we indicate with x^* , u_1^* , u_2^* the optimal state and strategies, and with $h_i^*(t) = h_i(t, x^*, u_1^*, u_2^*)$, the expected payoff for the i -th player in the problem (1) will be written as follows:

$$K_i(0, x_0, u_1^*, u_2^*) = \mathbb{E} \left[\int_0^{T_i} h_i^*(t) dt I_{[T_i < T_j]} + \int_0^{T_j} h_i^*(t) dt I_{[T_i > T_j]} + \Phi_i(x^*(T)) I_{[T_i > T_j]} \right], \quad (2)$$

where $I_{[\cdot]}$ is the indicator function and $\mathbb{E}[\cdot]$ is the mathematical expectation.

The following propositions are proved.

Proposition 15 *The expected payoff (2) for the problem starting at $t = 0$ is given by:*

$$K_i(0, x_0, u_1^*, u_2^*) = \int_0^{\omega} h_i^*(\tau) [1 - F(\tau)] + \Phi_i(x^*(\tau)) f_j(\tau) (1 - F_i(\tau)) d\tau. \quad (3)$$

Proposition 16 Employing the form of the hazard functions $\lambda_i(t) := \frac{f_i(t)}{1-F_i(t)}$,

the Hamilton-Jacobi-Bellman equations read as:

$$-\frac{\partial W_i(t, x)}{\partial t} + W_i(t, x)[\lambda_1(t) + \lambda_2(t)] = \max_{u_i} [h_i(t, x, u_1, u_2) + \Phi_i(x(t))\lambda_j(t) + \frac{\partial W_i(t, x)}{\partial x} \phi(t, x, u_1, u_2)]. \quad (4)$$

Finally, an example which is a modification of the standard model of extraction (see [7]) is considered. It is completely discussed and its optimal feedback solution is exhibited.

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On a Mutual Tracking Block for the Real Object and its Virtual Model-Leader

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Keywords: *Feedback control, Nonlinear system, Extremal shift.*

The research is devoted to a feedback control problem of stochastic stable mutual tracking for motions of (1) a real dynamical object, and (2) some virtual computer simulated model-leader, under dynamical and informational disturbances. The control and disturbance actions in the model are determined by proposed random tests. To obtain solution to the considered problem we apply the so-called *extremal minimax and maximin shift conditions*. Theoretical results are illustrated by numerical simulations.

The dynamics of x -object is described by the vector ODE -- nonlinear in controls u and disturbances v :

$$\dot{x} = A(t)x + f(t, u, v) + h_{din}(t), \quad t_0 \leq t \leq \theta, \quad (1)$$

subject to restrictions:

$$u \in P = \{u^{[1]}, \dots, u^{[M]}\}, \quad v \in Q = \{v^{[1]}, \dots, v^{[N]}\}. \quad (2)$$

Here symbols M and N are given numbers. Symbol $h_{din}(t)$ denotes a random vector-function restricted by the following relations:

$$|h_{din}(t)| \leq H, \quad E\{h_{din}(t)\} \leq \delta_{din}, \quad t \in [t_0, \theta], \quad (3)$$

where H stands for a sufficiently large constant, δ_{din} is a small constant, and symbol E denotes mathematical expectation.

Let us choose a partition $t_k \in \Delta\{t_k\} = \{t_0, t_1, \dots, t_k < t_{k+1}, \dots, t_K = \theta\}$, where K is a large number, and consider the finite-difference equation for x -object:

$$x[t_{k+1}] = x[t_k] + (A(t_k)x[t_k] + f(t_k, u, v) + h_{din}(t_k))(t_{k+1} - t_k). \quad (4)$$

Together with a real x -object we consider the motion of an abstract w -model:

$$w[t_{k+1}] = w[t_k] + (A(t_k)w[t_k] + \sum_{i=1}^M \sum_{j=1}^N f(t_k, u^{[i]}, v^{[j]})p_i q_j + h_{din}(t_k))(t_{k+1} - t_k). \quad (5)$$

Here numbers $p_i, i = 1, \dots, M$, and $q_j, j = 1, \dots, N$, satisfy conditions:

$$p_i \geq 0, i = 1, \dots, M, \sum_{i=1}^M p_i = 1, \quad q_j \geq 0, j = 1, \dots, N, \sum_{j=1}^N q_j = 1. \quad (6)$$

We assume that the motion of w -model is simulated by a computer, implemented in a regulator, and considered as the "leader" (or "pilot") for the motion of x -object.

Further, we consider the case when position $\{t_k, x[t_k]\}, k = 0, \dots, K$, of x -object is estimated with some informational error $\Delta_{inf}[t_k]$, such that at each time moment $t_k \in \Delta\{t_k\}$ only the distorted position $\{t_k, x^*[t_k]\}$ is known, where

$$x^*[t_k] = x[t_k] + \Delta_{inf}[t_k]. \quad (7)$$

Here $\Delta_{inf}[t_k]$ is a random vector.

Control actions for x -object and w -model, which provide mutual tracking in the combined process x -object, w -model-leader, are constructed as follows.

At the moment $t_k, k = 0, \dots, K-1$, a vector of actions $u^0[t] = u^0[t_k] \in P$, $t \in [t_k, t_{k+1})$, for the real x -object is chosen by probability test:

$$P(u^0[t_k] = u^{[i]} \in P) = p_i^0, \quad i = 1, \dots, M. \quad (8)$$

Here symbol P denotes probability and probabilities $p_i^0 : p_i^0 \geq 0, i = 1, \dots, M$, $\sum_{i=1}^M p_i^0 = 1$, are chosen from the so-called *Extremal Minimax Shift Condition*:

$$\begin{aligned} \min_{p^0} \max_q \langle l^*[t_k], \sum_{i=1}^M \sum_{j=1}^N f(t_k, u^{[i]}, v^{[j]}) p_i q_j \rangle = \\ \langle l^*[t_k], \sum_{i=1}^M \sum_{j=1}^N f(t_k, u^{[i]}, v^{[j]}) p_i^0 q_j^* \rangle, \end{aligned} \quad (9)$$

under restrictions (6). Here $l^*[t_k] = x^*[t_k] - w[t_k]$.

Let the "control action" $q^0[t_k]$ for the virtual w -model be chosen from the *Extremal Maxmin Shift Condition*:

$$\begin{aligned} \min_q \max_{p^0} \langle l^*[t_k], \sum_{i=1}^M \sum_{j=1}^N f(t_k, u^{[i]}, v^{[j]}) p_i q_j \rangle = \\ \langle l^*[t_k], \sum_{i=1}^M \sum_{j=1}^N f(t_k, u^{[i]}, v^{[j]}) p_i^* q_j^0 \rangle. \end{aligned} \quad (10)$$

Probabilities $\{q_j\}$ that define the stochastic disturbances $v[t_k] \in Q$ on x -object, and "actions" $\{p_i\}$ for w -model may take arbitrary values subject to conditions (6).

Under described above choices (9) and (10) of the random actions $u^0[t_k]$ for x -object and "actions" $q^0[t_k]$ for w -model, for any chosen beforehand numbers V^* and $0 < \beta < 1$, there exist sufficiently small numbers $\delta_0 > 0$, $\delta_{inf} > 0$, $\delta_{din} > 0$, $\delta > 0$, such that the following inequality holds:

$$P(V(t, l[t]) \leq V^*, \forall t \in [0, \theta]) \geq 1 - \beta, \quad (11)$$

if $l[t_0] \leq \delta_0$, $E\{|l(t) - l^*(t)| | l(t)\} \leq \delta_{inf}$, for any admissible $l[t] = x[t] - w[t]$, $t \in [0, \theta]$, $E\{h_{din}\{t\}\} \leq \delta_{din}$, and $\Delta t = t_{k+1} - t_k \leq \delta$. Here

$$V(t, l[t]) = V(t, x[t], w[t]) = |x[t] - w[t]|^2 e^{\lambda t} \quad (2)$$

Presented results are illustrated by a model example and its numerical simulation.



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Bidding Games with four Sequential Values of Share Price

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Keywords: *Bidding game, Incomplete information, Linearity domain, Vector payoff*

The Model. De Meyer and Saley (2002) introduced a model of multistage bidding between two agents for risky assets (shares). A liquidation price of a share depends on a random "state of nature". Before the bidding starts a chance move determines the "state of nature" according probability distribution p and therefore the liquidation value $C(p)$ of a share once for all. Both players know distribution p . Player 1 is informed on the "state of nature" and knows the share price $C(p)$. Player 2 is not. Player 2 knows that Player 1 is an insider.

At each subsequent step $t = 1, 2, \dots, n$ both players simultaneously propose their prices for one share. The maximal bid wins and one share is transacted at this price. If the bids are equal, no transaction occurs. Each player aims to maximize the value of his final portfolio (money plus liquidation value of obtained shares). The authors reduce the model of n -stage bidding to the zero-sum repeated game with lack of information on one side and solve the game.

In the model of De Meyer and Saley arbitrary bids are admissible. We investigate a discrete variant of the model: only integer prices and bids are admissible. This requirement is natural enough: it corresponds to having a minimum unit of exchange. We get the solution for these games of unlimited duration in Domansky, Kreps (2009), but the solution of n -stage games is an open problem.

Here we give solutions for n -stage games with four states of nature $0, 1, 2, 3$. So the model of n -stage bidding is reduced to the zero-sum repeated game $\Gamma_n(p)$, $p = (p_0, p_1, p_2, p_3)$, with lack of information on one side where p_i is the probability of state i , $i = 0, 1, 2, 3$. The game $\Gamma_n(p)$ is determined by four matrices of one-stage

payoffs of Player 1. For any distribution p we construct solutions of the n -stage games $\Gamma_n(p)$ on the base of our previous results on solutions of games of unlimited duration $\Gamma_\infty(p)$ and of games $G_n^3(p)$ with two states of nature 0, 3.

The previous results. It is well known that the value of repeated games with lack of information on one side is a piecewise linear continuous concave function over the set of distribution p . Optimal strategy of uninformed Player 2 depends only on the linearity domain containing p . The value function is determined by the linearity domains and its gradients -- vector payoffs for optimal strategies of Player 2: the value function at p is equal to the scalar product of p and the vector payoff for the linearity domain corresponding to p .

1. According to Domansky, Kreps (2009) the value function $V_\infty(p)$ of the four-state game $\Gamma_\infty(p)$ of unlimited duration has three linearity domains $D(0)$, $D(1)$ and $D(2)$ over the three-dimensional simplex Δ^3 . For $p \in D(k)$ the integer part of share price expectation $E[C(p)]$ is equal to k , i.e.

$$D(k) = \{p : k \leq p_1 + 2p_2 + 3p_3 \leq k+1\}, k = 0, 1, 2.$$

For Player 2' optimal strategy the four-dimensional vector payoff

$$\bar{\beta}(k) = (\beta^0(k), \beta^1(k), \beta^2(k), \beta^3(k)), k = 0, 1, 2,$$

corresponds to linearity domain $D(k)$: $\bar{\beta}(0) = (3, 1, 0, 0)$, $\bar{\beta}(1) = (1, 0, 0, 1)$ and $\bar{\beta}(2) = (0, 0, 1, 3)$.

2. If $p_1 = p_2 = 0$ the game $\Gamma_n(p)$ reduces to the game $G_n^3(p)$ ($p = p_3$) with two states of nature 0, 3 and with three reasonable bids 0, 1 and 2. In Kreps (2009) the explicit solutions for the games $G_n^3(p)$ are calculated by means of the second-order recursive sequence

$$\delta_{n+1} = 2(\delta_n + \delta_{n-1}), \delta_0 = 0, \delta_1 = 2.$$

The value function $V_n^3(p)$ on the interval $[0, 1]$ has four linearity intervals $I(0) = [0, 1/3]$, $I_n(1) = [1/3, p_n]$, $I_n(1') = [p_n, 2/3]$ and $I(2) = [2/3, 1]$, where

$$p_n = (\delta_{n-1} + \delta_n) / (\delta_{n-1} + 2\delta_n).$$

The four depending on n two-dimensional vector payoffs $\bar{\alpha}_n(\cdot) = (\alpha_n^0(\cdot), \alpha_n^3(\cdot))$ are:

$$\bar{\alpha}_n(0) = (0, 3 - 2/\delta_n); \bar{\alpha}_n(1) = (1 - 1/\delta_n, 1);$$

$$\bar{\alpha}_n(1') = (1 + 1/\delta_{n-1}, 1 - 1/\delta_{n-1}); \bar{\alpha}_n(2) = (3 - 1/\delta_{n-1}, 0).$$

Results. The value function $V_1(p)$ of one-stage game $\Gamma_1(p)$ with four sequential states of nature has five linearity domains on the simplex Δ^3 . The value function $V_n(p)$, $n \geq 2$, of the n -stage game $\Gamma_n(p)$ has seven linearity domains. As $n \rightarrow \infty$ these domains converge to the above described domains $D(0)$, $D(1)$, $D(2)$ and $\lim_{n \rightarrow \infty} V_n(p) = V_\infty(p)$.

Linearity domains of $V_n(p)$ and corresponding vector payoffs.

I. Four linearity intervals of function $V_n^3(p)$ over $[0, 1]$ generate four linearity domains $D_n(0)$, $D_n(1)$, $D_n(1')$, $D_n(2)$ over the simplex Δ^3 , where $D_n(0) \subset D(0)$, $(D_n(1) \cup D_n(1')) \subset D(1)$, $D_n(2) \subset D(2)$. If $n = 1$, the equalities $D_n(0) = D(0)$ and $D_n(2) = D(2)$ hold.

As number of step n grows these linearity domains shrink and their unification converges to the edge $\{p : p_1 = p_2 = 0\}$ as $n \rightarrow \infty$.

The four-dimensional vector payoffs $\bar{\beta}_n(\cdot)$ on domain $D_n(\cdot)$ has the first (for the state 0) and the forth (for the state 3) components coinciding with $\alpha_n^0(\cdot)$ and $\alpha_n^3(\cdot)$.

For calculating its second (for the state 1) and third (for the state 2) components $\beta_n^1(\cdot)$ and $\beta_n^2(\cdot)$ we write recursive sequences for these components. The obtained sequences δ'_n and δ''_n are of the second-order. They satisfy the same relations as δ_n but under other initial conditions. Taken into consideration the equalizing property of Player 2' optimal strategy and using backward induction we get

$$\begin{aligned} \beta_n^1(0) &= 1/3(1 + 2\frac{\delta_{n-1}-1}{\delta_n}), \beta_n^2(0) = 1/3(5 + \frac{\delta_{n-1}-4}{\delta_n}), \\ \beta_n^1(1) &= 1/3(1 + 2\frac{\delta_{n-1}-1}{\delta_n}), \beta_n^2(1) = 1/3(2 + \frac{\delta_{n-1}-1}{\delta_n}), \\ \beta_n^1(1') &= 1/3(1 + 2\frac{\delta_{n-2}}{\delta_{n-1}}), \beta_n^2(1') = 1/3(2 + \frac{\delta_{n-2}-1}{\delta_{n-1}}), \\ \beta_n^1(2) &= 4/3 + 2\frac{\delta_{n-2}-1}{\delta_{n-1}}, \beta_n^2(2) = 1/3(2 + \frac{\delta_{n-2}-1}{\delta_{n-1}}). \end{aligned}$$

II. Three linearity domains have the following form:

$$D(0) \setminus D_n(0), D(1) \setminus (D_n(1) \cup D_n(1')), D(2) \setminus D_n(2).$$

Note that if $n = 1$ the first and the third domains are empty.

The vector payoffs for the described domains coincide with the vector payoffs $\bar{\beta}(k)$, $k = 0, 1, 2$, of the game $\Gamma_{\infty}(p)$. This study was partially supported by the grant 10-06-00348-a of Russian Foundation of Basic Research.

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Periodicals in Game Theory

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Modernization Management as the Problem of Guaranteed Control-Estimation for Hierarchical System

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Keywords: *Modernization management, High-tech machinery plant, Guaranteed control-estimation, Hierarchical system*

Experience have been got by the author in process of consulting and retraining personal of innovative enterprises allows to assert that mathematical modeling of modernization management and risk control are of decisive importance. For such enterprises modernization is the only way of economical development and survival in sharp concurrence of high-tech market. For high-tech machinery plants it is common to work on forward contracts, fixing production volumes and prices of products for quite a long time (3-4 years). At the same time, inflation expectations cause the increase in cost of basic production factors. Rising prices for energy, raw materials, wages, utilities invariably bounds the profits. An efficient tool at the disposal of production managers is to reduce standard variable costs at the expense of upgrading and providing its administrative modernization. Chaotic effects of market behavior determine the relevance of research in terms of uncertainty. Interesting that long cycles of designing and manufacturing as a feature of engineering industries allow us to formulate an optimization problem within a guaranteed approach. In the paper the problems of modernization on different level of enterprise are discussed from the point of possible influence on cost – benefit scheme. The market competitiveness of machine-building plant depends on the level of productive capacity. However, the market competitiveness is mainly determined by the efficiency of the government and management for the production system and the whole plant.

That is why the problems of control and decision-making in organizational systems are of great significance and value. The mathematical formalism is motivated by

applied research and real-time modeling of team interaction, including the simulation of control of objects team motion and processes of decision making in modernization of high-tech engineering enterprises with restricted resources under hierarchically organized control.

The main statements are based on the notion of hierarchical system $(i)CG = (i)\{XG, PG, QG\}$, where a directed region $(i)XG = \{XG | \mathbf{c}\}$ is determined by comparison of inner boundaries for the whole system and combination of analogous regions connected with particular parts, subsystems. Description of organization is given by $(i)PG = \{PG, \Pi G\}$, where $(i)PG$ is a structural list of included subsystems of lower $(i-1)$ level $(i)PG = \{(i-1)CG.mj = (i-1)\{XG, PG, QG\}.mj\}$ and $(i)\Pi G$ is a structure matrix with integer values. A set $(i)QG = \{QG, \Theta G\}$, defines the generalization of a boarder of the system $(i)QG = \{(i-1)QG.ki\}$, where $(i-1)QG.kj = (i-1)CS(\eta_j, \eta_j')$ and $(i)\Theta G$ is an incidence matrix. Here:

$$1 \leq mj \leq Mm, \quad (i)\Pi G \in R^{Mm \times Mm}, \quad (i)\Pi G(mi, mj) = -1 \vee 0 \vee +1, \\ (i)\Theta G \in R^{Mm \times Mm}, \quad (i)\Theta G(ki, kj) = -1 \vee 0 \vee +1$$

The motion of participants may be treated in terms of system trajectories reflecting state, conceptual and organizational structures, results of observation and control. Participants may change positions in accordance with consequent decisions step-wise formed in accordance with positions on discrete grid. Hence a common interaction may be split into multiple layers of relatively independent processes for pairs of symmetrical systems.

The proposed model allows us to assert the priority of measures to modernize the management in comparison with purely technological improvements in the productive capacity of the enterprise. The assertion is based on axiomatic description of hierarchical systems, the choice of adequate statements of optimization problems under uncertainty for such systems, the study of analogues of the basic structural properties (duality, separation) that are obtained earlier for the guaranteed control and estimation problems in the operator form.

Models presented above provide a unified description of organizational structure, trajectories and cartographic information. They give possibility to describe in unified form a number of key assumptions for adequate mathematical modeling of high-tech machinery plants.

1) Situational scheme describing the interaction of open systems with relatively constant environment and participation.

2) Description of systems interaction in discrete time, that implies concerted shifts of an action of the participants.

3) Hierarchy in description of the participation, structure and behavior of the systems leads to the absorption of the description of lower layer subsystems that are out the observability boundaries.

4) Internal information model, describing data available, is constructed via inverse scheme and reflects shifts of perception with the center on the image of system.

The presented results are used extensively when working with students and training specialists in the field of production modernization of high-tech innovative engineering plants and preparing the management reserve.

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An Example of Using Game-Theoretic Models and Cognitive Maps to Analyze a Conflict of Interests between Russia and Norway in Barents Sea

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Keywords: *Game on linear cognitive map, Conflict of interests in Barents Sea, Dominant strategy.*

Abstract. A game-theoretic model on cognitive map is considered in the paper. The results are illustrated by the analysis of conflict of interests between Russia and Norway at the signing of the Treaty on maritime delimitation and cooperation in the Barents Sea in two. The system of beliefs, controls and goals of each side are formalized by cognitive map. Analysis of the game on the cognitive map showed that, signing of the Treaty will be profitable for Russia in the short term, but not favorable in the long run. Key causal relationship in the cognitive map, which led to this result were detected.

1.Introduction

Cognitive maps were introduced by Axelrod (1976) to clarify and improve decision making process. A cognitive map is a weighted digraph-based mathematical model of a decision maker belief system about some limited domain. Cognitive map nodes correspond to situation concepts. Concepts are interpreted as variables those values may vary. Weighted edges are interpreted as direct causal links from one concept to another. Game-theoretic models of interactions between several agents at a dynamic system in the form of a situation cognitive map was considered by Novikov (2008), Kulivets (2010, 2011).

2.Description of problem

Norway and Russia have sovereign rights over shelf space in the Barents Sea, which includes: 1) the Russian continental shelf (the right of Russia), 2) the Norwegian continental shelf (the right of Norway), 3) the offshore area of Svalbard (the right is governed by the Svalbard Treaty in Paris, 1920) and 4) continental shelves space

disputed zone. Disputed territory is about 175 thousand sq. km. Disputed area after 40 years of negotiations was divided into two approximately equal parts in Russian-Norwegian treaty on maritime delimitation in the Barents Sea on September 15, 2010 (hereinafter the Treaty).

Based on the materials from open source, with estimates of the situation in the Barents Sea and the Treaty, was constructed cognitive map representations of the situation surrounding the signing of the Treaty (see Fig. 1).

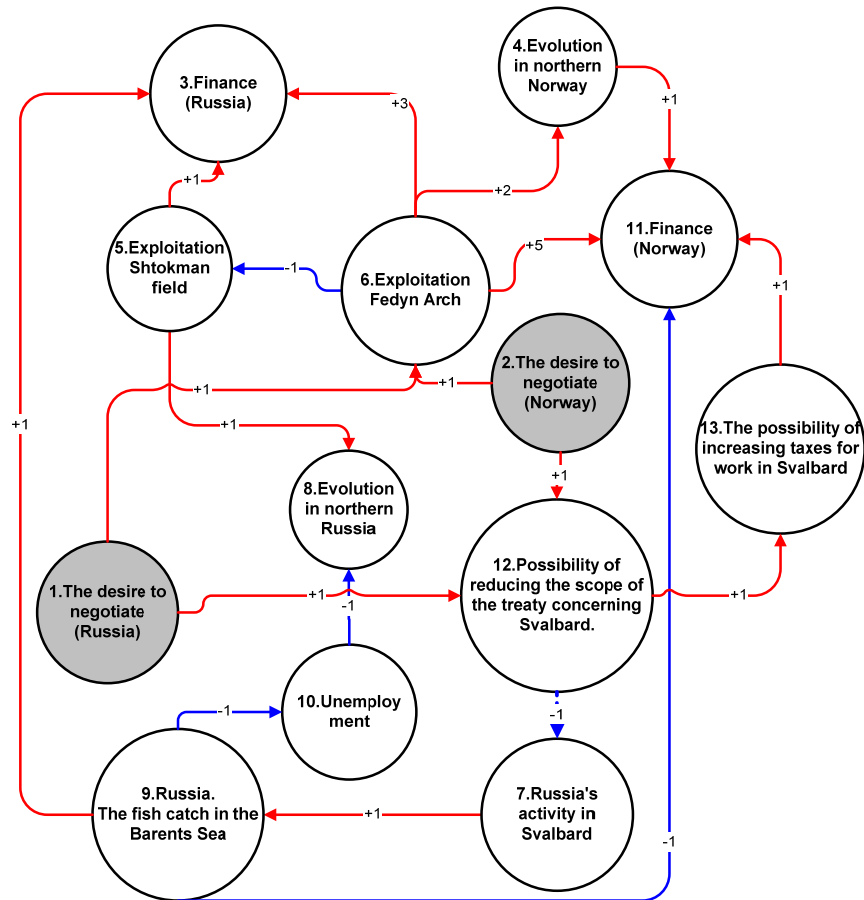


Fig.1. Cognitive map that reflects the causal links between concepts in the problem of the disputed territory in the Barents Sea. (The target concepts to Russia is #3 and #8, to Norway is #4 and #11)

Control concept for Russia - concept #1, for Norway - concept #2. The initial impact +1 for each of these concepts is interpreted as a desire to conclude the Treaty. The impact -1 as the absence of such aspirations, and on the contrary, his rejection. The impact value equal to zero, can be interpreted as indifference of the gamer on this issue. The target concepts for Russia will consider two: #3 and #8 for Norway #4 and #11. The

solution of the game is the equilibrium with dominant strategies. A set of solutions were found for different target time. Analysis of the game on the cognitive map showed that, signing of the Treaty will be profitable for Russia in the short term, but not favorable in the long run. Key causal relationship in the cognitive map, which led to this result were detected.

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МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

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The Strategy of Tax Control in Conditions of Possible Mistakes and Corruption of Inspectors

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Keywords: *Tax control, Corruption, Ineffective tax audit, Hierarchical game*

A generalization of the game-theoretical model of tax control adjusted for possible corruption and inspectors' mistake is considered. The hierarchical model, based on the game [5], has a three-level structure: at the highest level of a hierarchy is an administration of tax authority, in the middle is an inspector, subordinated to tax administration, and at the lowest level are n taxpayers. As in previous models [1--7], it is supposed, that an interaction between risk-neutral players of different levels of a hierarchy corresponds to scheme <<principal-to-agent>>.

The model is studied for the case when the penalty is proportional to the level of evasion, i.e., when the evasion is revealed, the k -th taxpayer must pay $(t + \pi)(i_k - r_k)$ where i_k and r_k are his true and declared incomes, $k = \overline{1, n}$, t and π are the tax and the penalty rates correspondingly.

The tax authority sends an inspector for the tax audit with the probability p_k , which costs c_k . A tax inspector may turn out a bribe-taker or make ineffective tax audit, i.e. make a mistake and don't reveal an existing tax evasion.

First, let's consider a case of corruption. In this case a tax control supposed to be effective, i.e. reveals existing tax evasions always.

For the bribe b_k audit inspector can agree not to inform his administration about the evasion revealed. With the probability \tilde{p}_k the tax administration makes re-auditing of the taxpayer, which is also absolutely effective, corruption-free and costs \tilde{c}_k . If a result of re-auditing is the revelation of the tax evasion concealed by the inspector, the taxpayer must pay $(t + \pi)(i_k - r_k)$ (as earlier) and the inspector must pay a fine

$$F = f \cdot (i_k - r_k),$$

where f is an inspector's penalty coefficient. As in [4], it is supposed, that the fact of corruption is very difficult to reveal and an inspector is punished only for negligent audit.

In the case of ineffective auditing it is assumed that the tax inspector can mistake and miss an existing evasion with the probability μ . The variable μ can be considered as a part of negligent inspectors of their total number. As in previous case, the tax administration makes re-auditing with probability \tilde{p}_k , which depends on μ . The negligent inspector pays a fine F and the tax evader pays penalty.

For every possible situation the players' profit functions and optimal strategies are found.

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Search with Store Chains

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Keywords: *Sequential Consumer Search, oligopoly, Noncooperative Games Theory.*

When a consumer searches sequentially for a product, the chances to turn up in a certain store chain is affected by the number of the stores this chain has. This paper takes the Stahl search model, and looks what happens when sellers have chains of varying size. The paper shows that no symmetric NE can exist when sellers are store chains of different sizes. The paper points out the asymmetric NE in the original model, and then, adding the chain size parameter and finds the NE of the search model with chain stores. We find that a chain with more stores will charge a higher price. Moreover, the expected profit per store is equal for all sellers. Similarly to the original equilibrium, all offers are weakly below the reserve price and searchers buy at first visited store. The results can explain a tendency that a more common store charges higher prices, and allow to use the Stahl search model in a more extended environment.

Introduction

Consumer search model can be used in a variety of fields, such as labour search or cheaper price search. There people sample prices and purchase where the good is cheaper. The field has a very developed literature, and it describes many economic phenomena, such as price dispersion. One of the simplest, yet the more realistic models was introduced by Stahl, in [3]. The model looks on a situation with a finite number of sellers offering an identical good and post prices simultaneously at the start of the game. The consumers are of two kinds - one type informed of the offered prices (shoppers) and one is not informed (searchers). The latter searches sequentially, when each search beyond the first one bears a positive cost.

There are clear asymmetries among sellers in the real world. One of the more important of those is the number of stores a seller has. This can affect greatly

the pricing and searching in the model. For example, in the paper by L.Z. Bakucs and I. Ferto [2], the various retailers have different size – from a few supermarkets in all of Hungary, up to a chain with several thousand stores. The prices for a very homogeneous product, milk in this case, vary between the stores. In [2] the data suggests a tendency that a more common store has a higher price, as depicted from their data in the table below (prices are taken from Table 2 in [2]).

Chain Name	Stores Number in Hungary ¹	Avg. price of milk
InterSpar	200-4004	182 HUF
Cora	Below 20	198 HUF
Match	Below 20	200 HUF
Tesco	200-400	205 HUF
Auchan	Below 20	211 HUF
CBA	Above 500	213 HUF
Plus ²	100-200	230 HUF
COOP	Above 500	240 HUF

Table 1 – Average milk price in Hungary and the number of stores each chain has. Price data taken from [2].

Looking at the table, one should consider whether the number of stores can be correlated with the price a retailer sets for a product. However, despite wide literature and a large number of extensions of the Stahl model, the literature dealing with asymmetries among sellers is very small. One of the few papers, by Astorner-Figari, and Yankelevich in [1] deals with a model, where there are two sellers and the search is not uniform among them. They come up with asymmetric NE, where the sellers select different strategies. As shown above, it is an important aspect which does exist in the real world. Therefore, this paper extends the discussion on asymmetries in the Stahl model into the general case of N sellers. The contribution of this paper is a general theoretic background to such model, existence and properties of asymmetric NE. The paper deals with the model with single store sellers, and also when sellers have a varying number of stores. It provides an important insight on how sellers of different size set prices. In order to easily compare the results to the existing literature the paper makes a reserve price

¹ I obtained only the current values. However, from it one can estimate the different number of stores in the past. This is due to the relatively big differences of the chains stores number.

² Plus stores were bought by InterSpar, after the relevant time period in [2]

assumption for consumers. An additional assumption is that the smallest seller is not unique.

Model

The original Stahl model, as introduced in [3], is as follows: There are N sellers, selling an identical good. The production cost is normalized to 0, and the seller can meet the demand. Additionally, there are consumers, each of whom wishes to buy a unit of the good. The mass of consumers is normalized to 1. This implies that there are many small consumers, each of which is strategically insignificant. The sellers are identical, and set their price once at the start of the game. If the seller selects a mixed strategy then the distributions are selected simultaneously (together with the pure strategy pricing), and the realizations take place only later.

The consumers are of two types. A part of consumers are shoppers, who know where the cheapest price is, and buy at the cheapest store. The rest are searchers, who sample prices. Sampling price in the first, randomly and uniformly selected, store is free. If the price there is satisfactory - the searcher will buy there. However, if the price is not satisfactory - the searcher will go on to search sequentially, in additional stores, where each additional search has a strictly positive cost. The second (or any later) store is randomly and uniformly selected from the previously unvisited stores, and the searcher may be satisfied, or search further on. When a searcher is satisfied, or visited all stores, she has a perfect recall. This implies she will buy the item at the cheapest store she had encountered.

Asymmetric Search Model

Suppose that the search now is among store chains, and not among single stores. Additionally, the price is identical in all the branches of the store chain, for example due to advertising of the store chain. The number of stores chain i has is λ_i , and this should have an effect on the search. Namely, the search is now not uniform, but according to the propensities λ . The definition of the model is the same as above, except replacing the underlined 'uniform' with 'according to the propensities λ '. For example, if a chain has two stores, each time a search is conducted this chain will attract twice the searchers the single store chain does. This reflects the probability to stumble upon a store randomly, yet due to the different chain sizes the search is not uniform among the sellers. Instead, the search is uniform among the stores.

Results

For both models (same size sellers and different size sellers) exists a multitude of asymmetric NE. Many of them can be constructed from symmetric NE of the original Stahl model by small adjustments – namely, where some sellers select purely the reserve price. Moreover, in the asymmetric search model (where sellers have different sizes) no symmetric NE exist and only the smallest sellers may offer prices below the reserve price. The symmetric search model NE has a defined structure, where the strategies do not differ too much among sellers. In such NE there are three possibilities for a seller: select the reserve price purely, select the price according to a (parameters and strategies dependant) common distribution F , or set a cut-off price p below which the seller selects price according to F , and above the seller selects only the reserve price with an atom. Additional characteristics of the NE are that the sellers have the same profit per store, and all searchers buy in the first store they visit. Note that in order to compare the results to the existing literature, the paper concentrates on NE with a reserve price. Additionally, the paper deals with the case where the smallest seller is not unique and, therefore, the number of sellers is at least 3.

Discussion

The reason for the lack of symmetric NE is quite easy to explain. Offering a discount beyond the reserve price has 2 effects: on the one side, it reduces the profit from the searchers that visit your store. On the other side, it increases the chance to be the cheapest seller and attract all the shoppers. As the mass of the shoppers is given, the second effect is common to all sellers. However, the first effect increases the larger the seller chain is. Therefore, highest incentives are among the smallest sellers. Due to the competition among the smallest sellers other sellers do not bother to enter the ‘shoppers market’, and sell only for the searchers at their reserve price.

The simplest example of an Asymmetric NE is of 3 sellers, where one selects the reserve price as a pure strategy, and the other sellers select strategies of the symmetric NE with 2 sellers and a smaller fraction of searchers (all those who do not visit the pure strategy seller). If the sellers have different sizes, the largest seller is the one setting the reserve price as a pure strategy.

These results adjust the expected empiric results done with the Stahl model. Firstly, the weight of the reserve price should be higher than depicted by the symmetric NE. Similarly, the discounts below the reserve price would be smaller.

Additionally, as seen for example in [2], there should be a positive correlation between the number of stores and the price in the chain. Lastly, there should be a correlation between the number of stores in the seller's chain and the profit she has. To sum up, the theory suggests a connection between the price a store chain set and the number of stores it has, and an additional explanation to empiric results of the model.

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Journals in Game Theory

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On the A-Equilibria Properties in N-Person Multicriteria Games

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Keywords: *Multicriteria games, Equilibria, Time-consistency.*

Using so-called A-optimality concept for vector-valued maximization [5], we propose a refinement of (weak) equilibrium set in n -person multicriteria games and discuss some properties of this optimality principle.

Consider n -person games where the player's payoff is given by a vector instead of a scalar (so-called multicriteria games). Pure strategy profile $\varphi = (\varphi_1, \dots, \varphi_n) \in \prod_{j=1}^n \Phi_j$ provides to each player i "payoffs" given by an $r(i)$ -vector valued function $H_i: \prod_{j=1}^n \Phi_j \rightarrow R^{r(i)}$, i.e. player i takes $r(i)$ criteria $H_i(\varphi) = (H_{i1}(\varphi), \dots, H_{ir(i)}(\varphi))$ into account. We denote by $MG(n, r(1), \dots, r(n))$ the class of all n -person multicriteria games.

For all $x, y \in R^t$ we will use the notation $y > x$ if and only if $y_i > x_i$ for all $i \in \{1, \dots, t\}$. The strategy profile $\hat{\varphi} = (\hat{\varphi}_1, \dots, \hat{\varphi}_n)$ is called (weak) equilibrium [4, 1] in multicriteria game $\Gamma \in MG(n, r(1), \dots, r(n))$, iff

$$\forall i \in N \quad \neg \exists \varphi_i \in \Phi_i: H_i(\varphi_i, \hat{\varphi}_{-i}) > H_i(\hat{\varphi}_i, \hat{\varphi}_{-i})$$

The set of all equilibriums in multicriteria game Γ denote by $ME(\Gamma)$.

Some properties of ME in finite n -person extensive multicriteria games with perfect (or incomplete) information [3] were discussed in [2].

One can use A-optimality concept [5] for reasonable refinement of ME set in n -person multicriteria game $\Gamma \in MG(n, r(1), \dots, r(n))$.

Let $A^k = (a_{ij}^k)$ be the $r(k) \times r(k)$ player k matrix with positive elements.

The strategy profile $\bar{\varphi} = (\bar{\varphi}_1, \dots, \bar{\varphi}_n)$ is called A-equilibrium in multicriteria game $\Gamma \in MG(n, r(1), \dots, r(n))$ iff

$$\forall k \in N \exists \varphi_k \in \Phi_k: A^k H_k(\varphi_k, \bar{\varphi}_{-k}) > A^k H_k(\bar{\varphi}_k, \bar{\varphi}_{-k}).$$

The set of all A-equilibriums in multicriteria game Γ denote by $ME^A(\Gamma)$.

Note that $ME^A(\Gamma) \subset ME(\Gamma)$. Let

$$\lambda = (\lambda_1, \dots, \lambda_{r(k)}) \in \Lambda^{r(k)} = \{\lambda \in R^{r(k)} \mid \lambda_j \geq 0, \sum_{j=1}^{r(k)} \lambda_j = 1\},$$

$$\mu_j(\lambda) = \lambda_1 a_{1j} + \lambda_2 a_{2j} + \dots + \lambda_{r(k)} a_{r(k)j}, \quad j = \overline{1, r(k)}.$$

Lemma. Given $\varphi_{-k} \in \prod_{j \neq k} \Phi_j$ let exist such $\lambda \in \Lambda^{r(k)}$ and $\bar{\varphi}_k \in \Phi_k$ that

$$\sum_{j=1}^{r(k)} \mu_j(\lambda) H_{k|j}(\bar{\varphi}_k, \varphi_{-k}) = \max_{\varphi_k \in \Phi_k} \sum_{j=1}^{r(k)} \mu_j(\lambda) H_{k|j}(\varphi_k, \varphi_{-k}).$$

Than

$$\exists \varphi_k \in \Phi_k: A^k H_k(\varphi_k, \varphi_{-k}) > A^k H_k(\bar{\varphi}_k, \varphi_{-k}).$$

Theorem. If the players' strategy sets $\Phi_k \in \text{Comp} R^{m_k}$, and payoffs functions $H_k(\varphi)$, $\varphi \in \prod_{k=1}^n \Phi_k$, are continuous, $k = \overline{1, n}$, than the multicriteria n -person game $\Gamma \in MG(n, r(1), \dots, r(n))$ possesses A-equilibrium (for every given $r(k) \times r(k)$ positive matrixes A^k , $k = \overline{1, n}$).

Corollary. Let $\Gamma \in MG(n, r(1), \dots, r(n))$ be a finite n -person multicriteria game (every players k has finite number m_k of pure strategies). Than the set of A-equilibriums in mixed extension of multicriteria game Γ is non-empty.

In addition, we discuss the problem of time-consistency [3] of A-equilibria in a finite n -person multicriteria extensive game Γ with perfect (or incomplete) information.

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Modeling Cloud Computing Auctions for Idle Capacity

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Keywords: *Cloud computing, Spot instance, Auction theory*

Here we discuss a new emerging auction in cloud computing field, which deals with unused capacity. The latter, if is not sold with additional restriction, will be wasted otherwise. Here we formulate the game theoretical (auction) problem and discuss its properties.

Introduction

Cloud computing is one of the newest and rapidly developing computing systems. Big companies foreseeing its benefits construct own private clouds (e.g., Amazon EC2, Google AppEngine, Microsoft Azure, etc). This emerged paradigms reveals a new yet to be fully understood market. As any market cloud computing provides commodities to be sold and bought. However, these commodities are distant representatives of conventional ones; thus, they have features which are not possible to have for traditional goods in ordinary markets. One of the most popular goods cloud operators sell is computational power¹. Here we will talk based on terms introduced by Amazon EC2², where minimal item for sell is called *instance* and charge scale is hour based. Thus, all the prices in terms of number instances of per hour. There are a plenty of instance types addressing different customer demands, but for simplicity we also restrict our discussion by one type. What is most important, is that there is a few selling policies for the instance, which makes them even more flexible for customers.

First of all, it is *reserved instances*, which are cheapest standard policies but they are bought on the terms of fixed minimum two year contract independently of how much the cloud is utilized. Second is *on-demand instances* -- the most expensive and

¹ We omit detailed discussion on technological aspects here.

² <http://aws.amazon.com/ec2/>

flexible policies, which have elastic properties for the clients, i.e., when customers need more resource they get it from the cloud immediately but paying much more for the reserve instances. The number of sold reserved and on-demand contracts give Amazon or any another cloud operator some estimates on daily/weekly picks; the operator guarantees 99.95% availability. The reliability includes possible spikes. The latter makes it necessary for cloud operators to have excessive set of data-servers to loaf around great portion of the time.

Thus, the third form for instances was introduced by Amazon - *spot instances*. The core concept is that at each moment of the time EC2 defines value of spot price SP and all the clients should announce own bids as well as how long they would like to run instances. Spot instances mechanism should address the following properties:

- At any moment of time all the bids which are greater than SP gains access to the cloud; if there were no access it is started.
- At any moment of time all the bids which are less than SP terminates from the cloud.
- Any instance terminated by EC2 have no need to pay for the last partial hour, otherwise it pays even for partial hour.

As we see spot instances are some form of auction mechanism, with additional (and probably novel) termination with no charge rule. Although the forms of auctions with bid/item withdrawals by bidder/seller exist, there are seems to be lack of auctions where partial use of items is possible and yet withdrawal exists.

In this work, we propose the problem statement as well as discuss the first most important properties of the auction. Finally, we will show some vulnerabilities issues related to the topic.

Minimax Confidence Intervals for Medium-Sized Samples

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Keywords: *Game theory, Statistics, Dynamic method.*

Though the game theory originated much later than statistics, it is the game theory that first introduced unified approach to solving statistical problems. Specifically, selecting appropriate loss function, we get basic mathematical statistics problems: hypothesis testing, interval or point estimation. However, herewith encountered statistical games prove to be computationally hard because of high dimensions. In our report we will show how to make interval estimate for parameter θ of discrete distribution via dynamic method of solving statistical games.

Interval estimation of unknown parameter θ for Binomial distribution $Bin(n, \theta)$ is a common problem in statistics. The most popular method of confidence interval (CI) construction is based on the fact that random variable (r.v)

$$Z = \frac{h - \theta}{\sqrt{(\theta(1-\theta))/n}},$$

with $h = x/n$

is close to normal $N(0,1)$ r.v. (Here x is observed value of r.v $Bin(n, p)$ and $h = x/n$ is observed success fraction). Thereby computed confidence intervals $[\theta_1, \theta_2]$ with limits

$$\theta_{1,2} = h \pm u_{1-\alpha/2} \sqrt{h(1-h)/n}$$

is commonly referred as Wald interval and is included in every statistics textbook. (u_α - normal distribution quantile).

Unfortunately, Wald interval does not provide confidence probability claimed, and, thus, wider and more complex Clopper-Pearson interval is used (CP). One method

of CP interval construction is implemented in Matlab program. It uses the following formulas for CP limits:

$$\theta_1 = \frac{x F_{\alpha/2}(m_1, m_2)}{n - x + 1 + x F_{\alpha/2}(m_1, m_2)}; m_1 = 2x; m_2 = 2(n - x + 1);$$

$$\theta_1 = \frac{x F_{\alpha/2}(k_1, k_2)}{n - x + (x + 1) F_{\alpha/2}(k_1, k_2)}; k_1 = 2(x + 1); k_2 = 2(n - x),$$

in which α is confidence probability and $F_{\alpha}(n_1, n_2)$ is Fischer distribution quantile with n_1, n_2 degrees of freedom.

Considering that CIs have unregular behavior (coverage probability oscillates depending on θ), there are continuing efforts to construct new CIs that are narrower than CP, yet with the same reliability [1].

The problem of parameter θ estimation with fixed width intervals is the most interesting, because only two easy interpretable values are used precision and reliability of estimate. In this case set of all acceptable CIs D_x , related to observation x , consists of identical intervals with fixed width Δ . Solution of thus encountered statistical game can be found via dynamic method devised by the author in [2].

Currently the program for fixed width CIs construction is developed by authors using the Matlab package. This program, given the number of observations n and discretization step nt and CI width θ finds, confidence probability γ . Besides, the program returns CI of width Δ for every value of x of observed r.v $Bin(x, \theta)$. Due to usage of dynamic method we solved the problem of interval estimation for $n \leq 200$, $nt \leq 2000$ for all Δ . Similar results were achieved [3] in Mathematics and Modelling academic department of Saint Petersburg State Transport University in 2001, using the minimax estimation system, but for $n \leq 50$ and $nt \leq 200$.

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The Shapley and Banzaf Values for Different Courses of Study

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One of the problems of the item response theory (test theory) is determination of test task weight [1]. In most tests the task weights are deemed equal, but as the complexity of test structure increases as well as the number of tasks in a test, necessity of test tasks complexity determination also increases. It is especially important if we want to compare the students who had different tests or if the time given for solving of all the test tasks is insufficient.

In order to assign (create) task weights of the test a cooperative game is designed. Value of characteristic function on any task subset of the test is a time required for preparation of a student to solve the tasks of this subset. In case of tree structure of the test a Shapley value and a Banzhaf power index are found.

In work [3] there was proposed a game theoretic approach for task weights assignment in test if a course of study is linear. Constructed there the Shapley and Banzaf values are similar to the values proposed by Littlechild S.C. and Owen G. A. [2] for "Airport game".

Definition 1. Let $I = \{1, 2, \dots, n\}$ be a set of test tasks. Complexity of test I is a nonnegative function v that assigns a number $v(K)$ (designated as a complexity of aggregate K tasks) to any task subset $K \subseteq I$ and possesses the following properties:

1. $v(\emptyset) = 0$.
2. For any two task aggregates K', K the following statement is true:
 $K' \subset K \Rightarrow v(K') < v(K)$.

If a student can solve all the tasks of the set $K \subseteq I$ then we say that he has a knowledge level K or he has a test solving skill K .

Thus, the complexity function v should be defined on 2^n subsets of set I , at that it is necessary to consider incomplete and sometimes inconsistent understanding of intuitive complexity. According to the game theory the complexity function v is a characteristic function of a cooperative game $\Gamma = \langle I, v \rangle$ possessing monotonicity property.

In practice, the function v is considered as an additive function and the complexity of any aggregate of tasks K , i.e. $v(K) = \sum_{i \in K} \phi_i$, where ϕ_i is a complexity (weight) of the i th test task, is determined using it.

One of the methods of task weight calculation in a test lies in the fact that the weight ϕ_i of task i is determined by a ratio N_i / N of students, solved this task, i.e. task weight is determined only after testing of a large group of students [1]. (Here N is a number of testees (test-takers), and N_i is a number of students who have solved the i th task.) However, such approach stimulates nonconventional methods of training: the less students have solved a task the more weight the task gets, but it is not improbable that the time spent on preparation of student to solving of more rare task may be not enough and a teacher will start to prepare students to solving of “rare” tasks breaking the logic of a teaching course.

In this work, we first define complexity function v and after we define items weights of the test I .

Let $v(K)$ be a time required for student to be prepared to solving of tasks of aggregate $K \subseteq I$. Then weight ϕ_i of the i -th task of the pedagogical test I is calculated by the following formula

$$\phi_i = \sum_{K \not\ni i} p_K \cdot (v(K \cup \{i\}) - v(K)), \quad (1)$$

where p_K is a probability distribution on a set of all the subsets $\{K\}$ which do not contain the i th task.

Since the difference $v(K \cup \{i\}) - v(K) \geq 0$ in the formula (1) is a time for getting i task solving skill at availability of skill to solve all the tasks from set $K \subseteq I$, then ϕ_i is a mathematical expectation of time for preparation of student to solving of the i th task of test for various knowledge levels.

If all courses of study have equal probabilities then $p_K = k!(n-k-1)!/n!$ $k = |K|$, and ϕ_i becomes an average time of student learning for equal probabilities of all courses of study. For this p_K the numbers ϕ_i are called the Shapley weights. (Term “course of study” means a sequence of teaching to test tasks I solving, i.e. a random permutation of numbers $1, 2, \dots, n$).

If we consider that all the knowledge levels are equally probable then $p_K = 1/2^{n-1}$ and ϕ_i becomes an average time of teaching of student, at equal probability of all the knowledge levels. In this case the numbers ϕ_i are called the Banzhaf weights.

Let's find the vectors of Shapley and Banzhaf for the tree graph of educational course structure. Let $I = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be the parts of course the knowledge of which are necessary to perform all tasks of the test. On the set I we introduce a partial order. We define the function of direct precedence $s : I \rightarrow I$. Thus, $s(\alpha)$ is the name of the section, after which it is possible to study the part α immediately.

Let α_1 be the name of the section from which a student start to study all parts of the set I . To simplify further designations we introduce following notations

$$s(\alpha) = \alpha'; \quad \alpha'' = (\alpha')'; \quad \alpha^{(k)} = \left(\alpha^{(k-1)}\right)'.$$

We suppose that the function of precedence s that defined on the set I has the following properties. The equation $\alpha' = \alpha$ on the set $I = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ has an unique solution α_1 . For any part $\alpha \in I$ there is an integer number k that following quality $\alpha^{(k)} = \alpha_1$ is true.

Definition 2. Let α, β be the course parts from the set $I = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$. We say that the part α is easier than the part β or part β followed α and write $\alpha < \beta$ if there is $k \in N$ that equality $\beta^{(k)} = \alpha$ is true.

We introduce the following notation

$$I(\alpha) = \{\beta \in I | (\beta < \alpha) \vee (\beta = \alpha)\}, \quad I_K = \{\beta | \beta < \alpha, \alpha \in K\} = \bigcup_{\alpha \in K} I(\alpha)$$

Let us determinate the characteristic function of a cooperative game by the following formula

$$v(K) = \sum_{\alpha \in I_K} a_{\alpha} \quad (2)$$

In the formula a_{α} is a time needed to student in order to solve the task α if he can solve the task from part $s(\alpha)$. Thus, $v(K)$ the time needed to student to study all parts from the subset K .

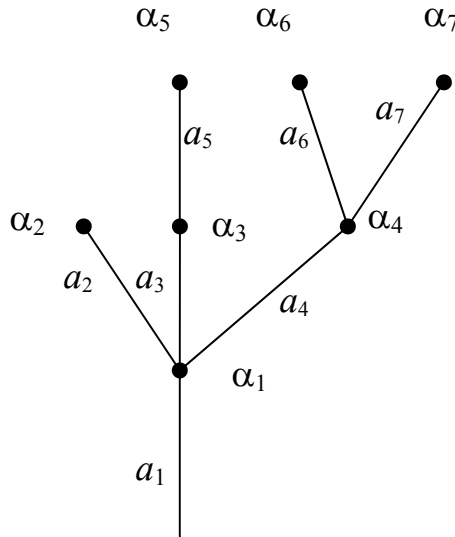
Theorem. If $I = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a set of course parts (tasks) of test I , $s : I \rightarrow I$ is a function of direct precedence of parts with the properties 1 and 2, α_1 is the name of the initial part, $a_{\alpha} = v(\alpha) - v(\alpha')$ is the time needed to student in order to solve the task α when he can solve the task from part $s(\alpha)$, $v(K)$ is the characteristic function of the game $\Gamma = \langle I, v \rangle$ defined by formula (2), then the Shapley and Banzaf values are given by formulas

$$\Phi_{\alpha} = \sum_{\beta \in I(\alpha)} \frac{a_{\beta}}{m_{\beta} + 1}; \quad \psi_{\alpha} = \sum_{\beta \in I(\alpha)} \frac{a_{\beta}}{2^m \beta} \quad (3)$$

with $I(\alpha) = \{\beta \in I | (\beta < \alpha) \vee (\beta = \alpha)\}$, $\bar{I}(\alpha) = \{\beta \in I | (\alpha < \beta) \vee (\beta \neq \alpha)\}$, $m_{\alpha} = |\bar{I}(\alpha)|$

Example. The test consists of 7 tasks. Its structure is defined by a graph

Figure 1. Test graph structure



(see Fig. 1). The numbers $a_i \geq 0$, $i = \overline{1,7}$ determine the time that is necessary for studying the part i under the condition that all the previous parts of course have been studied. According to the graph it is easy to construct a function of direct precedence s . It is given by the following table:

$$s = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 3 & 4 & 4 \end{pmatrix}$$

Let's find the value of Shapley and Banzafa. (We use the following abbreviations $\Phi_i = \Phi_{\alpha_i}$, $m_i = m_{\alpha_i}$, $\psi_i = \psi_{\alpha_i}$) To calculate them we find the quantity

$m_i = m_{\alpha_i}$ (number of tasks that are less than α_i)

$$m_1 = 6; m_2 = 0; m_3 = 1; m_4 = 2; m_5 = m_6 = m_7 = 0;$$

The components of the Shapley and Banzaf value can be found by formulas (3). For the example the Shapley value has the following components

$$\Phi_1 = a_1/7; \Phi_2 = a_1/7 + a_2; \Phi_3 = a_1/7 + a_3/2;$$

$$\Phi_4 = a_1/7 + a_4/3; \Phi_5 = a_1/7 + a_3/2 + a_5; \Phi_6 = a_1/7 + a_4/3 + a_6; \Phi_7 = a_1/7 + a_4/3 + a_7.$$

The components of the Banzaf vector are the following

$$\psi_1 = a_1/2^6; \psi_2 = a_1/2^6 + a_2; \psi_3 = a_1/2^6 + a_3/2; \psi_4 = a_1/2^6 + a_4/2^2;$$

$$\psi_5 = a_1/2^6 + a_3/2 + a_5; \psi_6 = a_1/2^6 + a_4/2^2 + a_6; \psi_7 = a_1/2^6 + a_4/2^2 + a_7.$$

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The Core and the Least Core in the non-Atomic Games

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Keywords: *Characteristic function, Core, Least core.*

Abstract: The existence of the core and the least core in the non-atomic games is proved.

Preliminaries.

Non-atomic games are considered in [1---6]

Let be $I = [0,1]$ the set of players, S --- σ -field of the measurable subsets of I (the coalitions), the measure μ on S , the characteristic function $V : S \rightarrow \mathbb{R}$, and $V = \mu$. The triple $\Gamma = \langle (I, S), \mu, V \rangle$ is called *non-atomic cooperative game*.

Let be \sim an equivalence relation on S defined as follow: for $s_1, s_2 \in S$ $s_1 \sim s_2 \Leftrightarrow \mu(s_1 \Delta s_2) = 0$. Denote by \tilde{S} the factor set of S for relation \sim , and the elements of \tilde{S} will be denoted by \tilde{s} and $\tilde{\mu}$ and $\tilde{V} : \tilde{S} \rightarrow \mathbb{R}$ --- the measure and the function generated by μ and V correspondingly.

In [5---6] on \tilde{S} is constructed a σ -field Σ and for each function $U : \tilde{S} \rightarrow \mathbb{R}$, such that $U = \tilde{\mu}$ is defined measure ν_U on Σ and integration in measure ν_U , such that

$$U(\tilde{s}) = \int_{\tilde{S}} \chi(\tilde{s}, \tilde{s}') d\nu_U(\tilde{s}'),$$

where

$$\chi(\tilde{s}, \tilde{s}') = \begin{cases} 1, & \text{if } \tilde{s}' \subset \tilde{s} \\ 0, & \text{otherwise} \end{cases}$$

In this report we shall use this definition.

Definition. The measure ν on \tilde{S} such that $\nu = \tilde{\mu}$ and $\nu(I) = V(I)$ is called *imputation*. The set of imputations will be denoted by M .

Definition of the finite subgame.

Let $\tilde{S}_i \in \tilde{S}$, $i = 1, \dots, n$ be a finite system sets and \tilde{S}^f be a generated field and \tilde{V}^f be a restriction of \tilde{V} on \tilde{S}^f . Let $p = (p_1, \dots, p_n)$ be an imputation in Γ^f . To this imputation p corresponds the set of imputations in M :

$$M^f(p) = \{v \in M \mid v(\tilde{s}) = p(\tilde{s}), \tilde{s} \in \tilde{S}^f\}.$$

Let $P = P(\Gamma^f)$ be the set of the imputations in Γ^f . Put

$$M^f = \bigcup_{p \in P} M^f(p).$$

The game $\Gamma^f = \langle \{\tilde{S}_1, \dots, \tilde{S}_n\}, \tilde{V}^f \rangle$ is called *the finite subgame of Γ*

Definition of the core. *The core of Γ is the set:*

$$C(\Gamma) = \{v \in M \mid v(\tilde{s}) \geq \tilde{V}(\tilde{s}), \tilde{s} \in \tilde{S}, v(I) = V(I)\}.$$

The core of Γ^f is the set:

$$C(\Gamma^f) = \{v \in M^f \mid v(\tilde{s}) \geq \tilde{V}^f(\tilde{s}), \tilde{s} \in \tilde{S}^f, v(\cup \tilde{S}_i) = \tilde{V}^f(\cup \tilde{S}_i)\}$$

Lemma. $C(\Gamma) \neq \emptyset \Leftrightarrow C(\Gamma^f) \neq \emptyset$ for every finite game.

Definition of the balanced system of the coalitions. Let be $\varphi: I \rightarrow I$ an automorphism and ν be its invariant probability measure such that $\text{supp } \nu = I$. Let be $s \in S$ a coalition such that $\nu(s) > 0$. Then $\{\varphi^k(s)\}$, $k = 0, 1, \dots$ be the balanced system because

$$\lim_{m \rightarrow \infty} \frac{1}{m} \sum_{k=0}^{m-1} \chi(x, \varphi^k(s)) = \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{k=0}^{m-1} \chi(\varphi^{-k}(x), s) = \int_S \chi(x, x') dN_s(s') = \int_I \chi(x, s) d\nu(x) = \nu(s),$$

where N_s be a measure on Σ generated by ν .

Theorem 1. For $C(\Gamma) \neq \emptyset$ it is necessary and sufficient to every automorphism φ its invariant probability measure ν and $s \in S$ to have

$$\lim_{m \rightarrow \infty} \frac{1}{m} \sum_{k=0}^{m-1} V(\varphi^k(s)) = \int_S V(s') dN_s(s') \leq V(I)\nu(s).$$

The least core.

Definition. For $v \in M$, $s \in S$ the difference $e(v, s) = V(s) - v(s)$ is called *the excess of the coalition s for the imputation v* .

Consider the following zero-sum two person game $G = \langle S, M, e \rangle$. The set of the optimal strategies of the player II is called *the least core*.

Theorem 2. *The game G has a solution.*

Corollary. *The least core exists.*

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The Tragedy of Commons, Resource Extraction and Price Dynamics

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Keywords: *Tragedy of commons, Resource extraction, Price dynamics.*

In absence of well defined property rights, the exploitation of natural resources is doomed to end up in the tragedy of the commons (Gordon, 1954; Hardin, 1968), i.e., excess extraction that may lead to exhaustion or extinction in finite time.

This is surely true for non-renewable resources, whose optimal exploitation rate is implicitly defined by the Hotelling rule (Hotelling, 1931), relating the inflation rate of the final product (whose production requires the exhaustible resource) to discounting. The standard approach to this problem makes use of the assumption of perfectly competitive markets (commonly appearing in textbooks, see e.g. Pearce and Turner, 1989; but also in recent research on the frontier of the discipline as in Dawid and Kopel, 1997; and Figuieres, C. and M. Tidball, 2010) or monopoly (Hörner and Kamien, 2004). We propose to depart from these two polar assumptions to treat the more representative case of an oligopoly, which, incidentally, has the desirable property of lending itself to deliver monopoly and perfect competition as limit cases.

In order to model the relationship between price dynamics, discounting and resource exploitation under strategic interaction, we insert the resource dynamics into a well known dynamic model of oligopoly with price adjustment dating back to Fershtman and Kamien (1987). The basic features of the resulting differential game in continuous time can be succinctly spelled out as follows.

Consider an industry in which the production of a final good requires a natural resource that can be exploited simultaneously by N firms (which, in the initial setting, are unregulated Cournot profit-seekers). At any $t \in [0, \infty]$, each of them supplies

quantity $q_i(t), i \in \{1, 2, \dots, N\}$, of the same homogeneous good at a total cost $C_i = cq_i(t) + zq_i^2(t)$. In each period, market demand determines the notional price level:

$$p(t) = a - \sum_{i=1}^N q_i(t)$$

In general, however, $p(t)$ will differ from the current price level $p(t)$, due to price stickiness, and the price evolves according to the following equation:

$$\dot{p} = s(\hat{p} - p)$$

Notice that the dynamics above establishes that price adjusts proportionately to the difference between the price level given by the inverse demand function and the current price level, the speed of adjustment being determined by the constant $s \in [0, \infty]$. This amounts to saying that the price mechanism is sticky, that is, firms face menu costs in adjusting their price to the demand conditions deriving from consumers' preferences: they may not (and, in general, they will not) choose outputs so that the price reaches immediately $p(t)$, except in the limit case where s tends to infinity.

The dynamics of the natural resource, whose stock at any time is $x(t)$, follows the equation:

$$\dot{x} = -\beta \sum_{i=1}^N q_i(t) + \delta x(t),$$

where $\delta > 0$ is the natural rate of reproduction (if the resource is renewable).

The model being linear-quadratic in (p, q_i) and linear in x , we can solve analytically the resulting Bellman equation to characterize the feedback (i.e., Markov perfect) equilibrium of the game. By doing so, we get a relationship between industry concentration, the speed of price adjustment, time discounting and the intensity of resource exploitation during the game (through the analysis of first order conditions) as well as in the steady state.

The basic setup can be easily extended to account for

- (1) Regulation through Pigouvian taxation
- (2) External effects (pollution) generated by production, itself liable to Pigouvian taxation (as in Benckroun and Long, 1998)
- (3) Intra-industry trade (as in Copeland and Taylor, 1994; and Beladi and Oladi, 2011, inter alia)

- (4) R&D for process innovation (lowering the technical coefficient ultimately determining the exploitation rate)

Additionally, the present model allows us to examine another relevant issue, namely, the determination of the optimal number of firms in an industry inherently affected by a tragedy of commons (see Cornes and Sandler, 1983; Cornes, Mason and Sandler, 1986; Mason and Polasky, 1997, *inter alia*). This aspect is, in principle, ambiguous as any increase in the population of firms brings about an output expansion which (i) has a desirable price effect, increasing consumer surplus, but (ii) puts more pressure on the natural resource itself, possibly leading to its quicker exhaustion. This is a way of addressing the crucial issue of intergenerational equity that goes hand in glove with the design of the efficient management of natural resources. That is, in much the same way as in the long standing debate on the so called Schumpeterian hypothesis concerning the relationship between market power and technical progress, also here we face a trade-off between static and dynamic efficiency which deserves a very close look, in view of its dramatic long run consequences.

At this stage, our view of the project leads us to expect a first paper to cover the basic aspects of the interplay between industry structure, price dynamics and time discounting in a single paper illustrating also the effects of Pigouvian taxation on resource exploitation, as specified in point (1) above (i.e., without environmental effects) and the matter of the optimal number of firms. A second paper (or more) would then address points (2-4).

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РОССИЙСКИЙ ЖУРНАЛ МЕНЕДЖМЕНТА

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Centrality in Weighted Social Networks. A Game Theoretic Approach.

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Keywords: *Weighted networks, Centrality, Probabilistic Myerson value, Generalized probabilistic Myerson value*

This communication deals with a key issue in network analysis: the problem of centrality of nodes in networks. Most of the work on this topic is about the case in which links are dichotomous. In the present work, we consider the case of networks in which ties are not just either present or absent, but have some weight attached to them. Following Granovetter (1973) we will interpret these weights as a function of duration, intimacy or intensity of the relations. Moreover, we will suppose that actors in the network are simultaneously players in a cooperative TU game representing the interests that motivate their interactions. Our approach is then a game theoretic one. We will propose as centrality measure for each node its probabilistic Myerson value (Calvo et al., 1999 and Gómez et al., 2008) assuming that the game is a symmetric one and thus, no a priori differences among players exist. We will prove that this measure satisfies some relevant properties to be considered as a centrality one.

Organizational Design under Externalities

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Keywords: *Organizational Design, Networks, Externalities, Hierarchies.*

In recent years the theory of organizational design has mostly focused on the efficiency and allocative properties of organizations, with an emphasis on their role in providing the right incentives to members, facilitating the information flows, coordinating their internal tasks and so on (Hart and Moore 2005, Harris and Raviv 2005, Alonso et al. 2008, Bandiera et al. 2009, Garicano 2010, among the others). If the organization is depicted as a network whose links represent the relations between its members, the optimal design of the network should attempt to maximize some objective-functions, as aggregate production, profit or any other index used to calculate the participants' payoffs (workers, shareholders, stakeholders, etc.).

However, no matter how efficient an organization may be, in order to function it has to meet certain stability properties. Stability is essential when organizational members have a choice of opting out of their organization, either individually or collectively. For instance, an industrial cartel needs to meet all claims of its members which would, otherwise, find profitable it to defect from it. In a large company, certain units may credibly threaten to set up hostile spin-offs. Within a party, certain coalitions may consider to jointly defect from the party to form a new political entity. Stability appear more of an issue when individual members (or subcoalitions) possess concrete outside options acting independently of (or against) the rest of the organization. When such options are substantial, the design of an organization may help to weaken such threats and, insodoing, contribute to the stability of the organization.

To fix ideas, think of an organization as a group of agents trying to make decisions on a set of issues (and think of each decision inducing a well defined welfare distribution among members). Suppose also that the members of the organization can object to a given decision by departing from the remaining members, to set up a new and possibly competing organization. A collective decision can, thus, be viewed as stable as long as there is no group of members within the organization who may find it profitable to reject the decision by acting as a separate organization. It is obvious that only stable decisions are viable, and that the existence of stable decisions is a requisite for the organization to be functioning as a decisional unit.

A first way in which the structure of the organization can affect stability is by defining the patterns of communication between members. Suppose in fact that two agents are able to communicate and coordinate actions if and only if they are linked in the network. Since joint rejections require some degree of coordination (agents must know of other agents' dissatisfaction of current distribution of payoffs, they must commit to jointly act against the organization, as well as monitor each other in doing so), a group of members can reject a given decision only if each of them is linked to at least another member of the group. In terms of the network, each pair of members of the rejecting group must be linked through a path that pass through the members of the group only. Since the network defines such groups, organizational design determines the set of possible rejections. A nice result by Demange (2004) shows that only hierarchical organizations guarantee the existence of stable decisions, by appropriately defining the set of potentially objecting groups.

The stepping stone of our paper is the observation that in many economic problems, the welfare that an objecting group can guarantee to its members heavily depends on what happens to the organization after the objection. Particularly so when objecting means defecting from the organization to set up a new one. Take, for instance, a set of firm considering defecting from a large cartel. Their profit expectations in doing so are obviously affected by the expected behaviour of the firms remaining inside the cartel. If these firms are expected to remain united in a smaller cartel, then defecting firms would envisage a duopolistic competition between two cartels. If the original cartel is expected to break apart, defecting firms would expect to compete against a large numbers of independent firms.

In general, the collapse of the original organization is beneficial to defectors whenever competition is more profitable against many independent agents rather than

against a united block of cooperating agents (in game theoretic terms, a case of negative spillovers). When the opposite is true (positive spillovers), defectors would prefer to face a united organization.

We claim that the sign of such spillovers can affect the design of stable organizations. More densely connected organizations are intrinsically more stable than sparse connected ones under negative spillovers, while the opposite holds for the case of positive spillovers. In particular, hierarchical organization, by definition not densely connected, may face severe stability issues under negative spillovers, and may instead prove stable under positive spillovers. Additionally, the distribution of payoffs within the organization is clearly affected by the sign of spillovers. For instance, in a star network, the central node will face very high outside options under negative spillover compared to peripheral nodes, simply because they expect opposite consequences in the event of a defection: the central node would disintegrate the organization, while peripheral nodes would face remaining nodes organized in a smaller but united organization.

How these considerations interplay with efficiency considerations is an interesting issue in organization theory, which has never, to our knowledge, been addressed within a formal theory. Conclusions will certainly depend on how the economic model behind the creation of value within the organization relates to the type of spillovers across competing organizations. As a consequence, optimal design may find it either possible or impossible to attain both efficiency and stability, and can be determined by how the tradeoff between efficiency and stability is solved, that depends, ultimately, on the designer's preferences.

Treasure Game

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Keywords: *R&D, Search, Uncertainty.*

In this article, we analyze a R&D race where the prize value is common knowledge, but the search costs are unknown *ex ante*. There are many examples of this situation in reality: pharmaceutical firms participating in a R&D race of drug discovery; journalists looking for a movie star in the city hotels; researchers looking for solutions of the six Millennium Prize Problems in mathematics.

There is a substantial recent literature on R&D. Loury (1979), Dasgupta and Stiglitz (1980a, b), Lee and Wilde (1980) amongst others assume that each firm in R&D competition makes once-and-for-all expenditure which determines the winner. The typical outcome of these models is that aggregate expenditure on R&D is too high relative to the monopoly outcome. In equilibrium firms overinvest, because each firm considers only its own marginal benefit from investment, and does not take into account negative externality it imposes on other firms. See Reinganum (1989) for more detailed discussion about this literature.

Reinganum (1981) extends the above literature by considering a dynamic R&D race where each firm chooses a time path of expenditures. The outcome is that if knowledge is a pure public good and the social value of the innovation is high enough, then firms under-invest as compared to the monopoly outcome. In contrast to the previous literature, therefore, aggregate expenditure on R&D may, depending on the exogenous parameters, be either too high or too low relative to the monopoly outcome. However, one of the critical assumptions Reinganum makes is that the success function is exponential. As a result, previously acquired knowledge does not change the probability of current success in the race; that is, the equilibrium strategies may be time-independent. There are many situations, however, where the memorylessness assumption

is not satisfactory. One such situation arises when the search domain, while potentially large, is finite. Think for example of journalists looking for a movie star in the city hotels, or private detectives searching for a criminal. The exponential distribution which is standard in the literature is not appropriate in such cases. This paper uses a uniform distribution instead.

In particular, we analyze a dynamic R&D model where a given number of players search for a treasure hidden somewhere on an island. The value of the treasure is common knowledge, and search is costly. Once the treasure is found the game ends. All players observe the current island size and make their search decisions simultaneously. If the treasure is not found in the current period, search in the next period occurs over the remaining unsearched area. We assume that during the search, players are informed about the areas that have already been searched by their opponents. In other words, we assume that knowledge is a pure public good, see Reinganum (1981). The game we consider has Schelling's (1971) "conflict of partnership and competition" property: players are naturally competing against each other each period, but each player benefits from the other players' previous periods' unsuccessful search, because it increases his chance to find the treasure in the current period.

We assume that players are searching different parts of the island, and only one player can obtain the treasure. If several players find the treasure simultaneously (search the same part of the island), each of them incurs costs; but the treasure will be destroyed (players do not get any treasure). This assumption is standard in the R&D literature (see for example Chatterjee and Evans, 2004). Intuitively, if several players discover the treasure simultaneously, a fierce competition between them runs down the surplus. A good example of such a situation for just two players is 1960's Lockheed and Douglas jet development competition. For more detail, see The Economist (1985); and Chatterjee and Evans (2004). Many examples of simultaneous discoveries in science can be found in Merton (1973).

We analyze a stochastic game in which each state is described by the remaining unsearched area. There are multiple subgame perfect equilibria in the game; we restrict our attention to the symmetric Markov perfect equilibria (SMPE). Imposing Markov perfection not only makes our analysis simpler, while still being consistent with rationality; but it also makes our results directly comparable to those in the previous literature. See Maskin and Tirole (1988) and Bhaskar et al.(2010) for a general discussion of why the use of SMPE is appropriate.

Among all SMPE, we only consider the efficient SMPE; that is, the equilibrium with the highest total expected payoff in the absence of collusion. We find that the efficient SMPE (for a fixed discount factor δ and a fixed number of players n) is always a spline of degree one. (A spline is a special function defined piecewise by polynomials; see for example Ahlberg, Nielson, and Walsh (1967).)

If $n \leq 2$, the efficient SMPE always exists. Our approach gives a complete characterization of the efficient SMPE in this case. If $n \geq 3$, we can characterize the efficient SMPE only for relatively small island sizes - the maximum number of search periods is two. We show by example that for larger island sizes where more than two periods of investment are required, neither SMPE is efficient.

We compare the efficient SMPE for $n \geq 2$ to the case of monopoly ($n = 1$). Relative to the latter, multi-player search is typically inefficient, except for very small islands when players behave as a cartel and search lasts just one period. In general there are two types of possible inefficiency. First, in the case of small islands, multiple players search too fast: the probability of finding a treasure is relatively high, which means players have an incentive to over-search in the current period. This is a standard tragedy of the commons effect. It leads to over-investment in comparison with the case of monopoly. Second, in the case of large islands, players undertake insufficient search: the probability of finding the treasure is relatively low, so the immediate payoff from search is negative. Players want others to search and incur current losses, hoping that the treasure will not be found in the current period. In other words, there is an incentive to postpone search to a future period, when it will be more profitable. This is a standard free riding effect. It leads to under-investment in comparison with the monopolist. Note that in the present model, both the tragedy of the commons and the free riding effect may endogenously arise within the same project. This contrasts to Reinganum (1981) where, because of the memorylessness assumption, only one of these effects is present for a single project.

Since in the efficient SMPE all players make the same decisions simultaneously, they have equal probabilities to find the treasure in any period. Therefore, it seems natural to conjecture that a smaller island (smaller costs) is better than a bigger island for all players. In fact, it turns out that this conjecture is incorrect. By example we illustrate that players might be worse off with a smaller island. This surprising observation, which we refer to as Puzzle 1, means that an increase in expected

costs might make all the players better off. Puzzle 1 has the following intuitive explanation. If the island is small, the tragedy of the commons effect is strong, and players over-search the island. If the island size is increased, the tragedy of the commons effect decreases, and players search the island more efficiently. It turns out that this efficiency improvement may be large enough to outweigh the increase in the cost of searching the larger island.

It also seems natural to anticipate that the expected number of search periods monotonically increases with the island size. As we show in the example, this conjecture is also incorrect. In fact a larger island can speed up or slow down the search process. We refer to this observation as Puzzle 2. This puzzle can be explained by players' inefficient behavior when the island size is relatively large and there are many players. Due to the free riding effect, players have a greater incentive to under-invest. When there are many players, this effect may be very strong, leading to non-monotonicity of the search function. For some parameter values, therefore, neither SMPE is efficient.

In the special case when the unsearched area of the island is exactly equal to the treasure value, the tragedy of the commons and free riding effects are absent, and search with $n \geq 2$ players reproduces the monopoly outcome. This happens when the discount factor is sufficiently low $\delta \leq \frac{1}{2}$, guaranteeing that players search the island in at most two periods. For this unique island size, players get zero expected payoff in period one. Consequently the objective function of players is to maximize their expected payoffs from the second period only. In the second period in the symmetric equilibrium, each firm receives a payoff proportional to the payoff of the monopoly. This guarantees that multi-player search reproduces the monopoly outcome.

In a separate strand of the R&D literature, Fudenberg et al. (1983) and Harris and Vickers (1985a, 1985b), amongst others, analyze a deterministic multistage race model where it is assumed that the firms transit from one stage to the next in a deterministic fashion. They investigate the question of when the firm that is behind in the race engages in catch-up behavior. The outcome is that the slightest advantage of one firm causes the other to drop out of the race. Similar to the spirit of this paper, Doraszelski (2003) extends this literature by developing a model of an R&D race which does not rely on the memorylessness assumption. The results are strikingly different

from the previous literature. In particular, under some conditions, the firm that is behind in the race engages in catch-up behavior.

Chatterjee and Evans (2004) analyze a R&D race using a model which is complementary to ours. In their framework, two competing firms observe the other's past choices and search strategically. These firms simultaneously choose between two research projects, where it is common knowledge that exactly one of these projects will be successful if enough investment is made. While agents in their model decide which area to search in (how much they search each period is exogenously determined), agents in our model decide how much to search (the location has no importance).

Fershtman and Rubinstein (1997) consider a number of inefficiencies that arise from multi-player search in comparison to single-player search. They develop an interactive model in which two players search for a single hidden treasure in one of a given set of labeled boxes. In contrast to our paper, they assume that there is a fixed cost to hiring a search unit, while opening a box is a free operation. Moreover, unlike in our paper, they also assume that all research findings may be privately retained.

Our paper is related to the literature on strategic experimentation with publicly observable actions and outcomes; see Bolton and Harris (1999, 2000), Keller, Rady and Cripps (2005), Keller and Rady (2010) and Klein and Rady (2011). They use the two-armed bandit framework to model the trade-off between experimentation and exploitation in teams. In particular, Klein and Rady (2011) assume negative correlation of the quality of the risky arm across players. Note that strategic interaction in their model arises out of purely informational concerns. In our context, they assume that a player benefits from the other player's previous periods of unsuccessful search. However, contrary to us, there is no pay-off rivalry among players in their model.

Differential Bargaining Games as Microfoundations for Production Function

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Keywords: *Bargaining, differential games, Production factors, Choice of technology, Duality, Production function*

It is well known that the acceptance of concrete types of production functions in economics, such as Cobb-Douglas and CES forms, was rather occasional and till now not enough attempts have been made to explain and justify the wide used types of production function – e.g. Matveenko, 1997, Jones, 2005, Nakamura, 2009, Matveenko, 2010, Dupuy, 2011. In the paper models resulting in the Cobb-Douglas production function are constructed on base of differential games of bargaining and by use of dual relations in production and distribution.

There is a duality relation between a physical side of economy (resources and institutions) and its institutional side (distributional relations between social groups). The idea of the models presented here is that the distributional behavior can be described by a differential game of bargaining.

The knowledge of any one of a pair of dual objects is enough for restoring the another one. The spreading of duality in economics is connected with the mathematical fact that convex closed sets (which can be often found in economic systems) can be described in two ways: by enumerating their elements (a primal description) and by enumerating closed subspaces containing the set (a dual description).

A dual relation between the institutional and the physical sides of the economy allows to achieve an independent description of production function on base of a differential game on an institutional side of the system.

Three differential games are proposed to describe a behavior of economic agents in processes of prices and weights formation. In the benchmark model of price bargaining players are interested in changing the same price in opposite directions. It is

shown that under some conditions this game leads to the Nash bargaining solution. The benchmark game is modified to games in which players are workers and capital-owners. They change (different) prices of their owned resources or change weights (moral-ethical assessments). One of these games describes bargaining of workers and capital-owners for their factor prices. In another game the same players bargain for the weights (moral-ethical assessments); these weights enter a Rawlsian-type criterion which is used further by an arbiter (community).

These games result in construction of structures (a price curve in one case and a weight curve in another) which are dual to the unit level line of the production functions. Ultimately, under constant bargaining powers of the participants, these games lead to the Cobb-Douglas form of production function.

One of the duality relations used in the paper corresponds to the representation of production function by use of the Euler theorem in an “extremal” form:

$$F(K, L) = \min_{p \in \Pi} px \quad (1)$$

which means that the production function represents a result of a choice of the price vector from the set Π . Another type of duality corresponds to a representation found by Matveenko (1997, 2010) and Jones (2005) which reminds (1) but uses the Leontief function as an inner product:

$$F(K, L) = \max_{\lambda \in \Lambda} \min \{ \lambda_K K, \lambda_L L \}$$

Here Λ is a technological menu which corresponds to the production function F . In the present paper both the usual and the generalized type of duality are used.

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Game-Theoretic Tender's Model and Reputation of Arbitrators

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Keywords: *Bargaining, Tender, Reputation, Optimal strategies.*

We consider a game-theoretic model of tender [1] in which two players submit the projects presented by two parameters. First player likes to maximize the sum $x+y$ and the second one - to minimize it. To determine the winner in tender the arbitrators are invited. The reputation of arbitrator is determined via reputation matrix A . Each arbitrator prefers a proposal which is most closer to her mind. The equilibrium in this game is derived.

For instance consider the symmetric case with two arbitrators and the opinions of the arbitrators are presented by gaussian distributions

$$f_1(x,y)=\frac{1}{2\pi}\exp\left\{-((x+c)^2+(y-c)^2)/2\right\}$$

$$f_2(x,y)=\frac{1}{2\pi}\exp\left\{-((x-c)^2+(y+c)^2)/2\right\},$$

where c is a parameter.

Using the reputation model we obtain the resulting distribution

$$f_a(x,y)=a_1f_1(x,y)+a_2f_2(x,y),$$

where a_1, a_2 - reputation of the arbitrators and $a_2=1-a_1$.

The optimal strategies of the players in the game depends on the reputation of the arbitrators. For first player that is $(\sqrt{\pi}+c(a_2-a_1), \sqrt{\pi}+c(a_1-a_2))$ and $(-\sqrt{\pi}+c(a_2-a_1), -\sqrt{\pi}+c(a_1-a_2))$ - for the second.

If the reputations of the arbitrators are equal then the components in the equilibrium coincide in absolute values. If the reputations are non-equal then component which corresponds to the arbitrator with largest reputation is greater than for the smallest one.

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Optimal Strategies in the Game with Arbitrator

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Keywords: *Game, Arbitration procedure, Equilibrium, Mixed strategies*

We consider a non-cooperative two players zero-sum game, related with arbitration procedure. The players make their offers and the arbitrator's decision is simulated by a random variable with odd possible values. The Nash equilibrium in this game in mixed strategies is found.

We consider a zero-sum game, in which two players, L and M , called respectively the Labour and the Manager, have a dispute on an emprovement in the wage rate. The player L makes an offer x , and the player M --- an offer y ; x and y are arbitrary real numbers. If $x \leq y$ there is no conflict and the players agree on a payoff equal to $\frac{x+y}{2}$. If, otherwise, $x > y$, the parties call in the arbitrator A . Denote the arbitrator's settlement by z . In the final-offer procedure [1] the arbitrator chooses the offer, which is closer to its solution z . The payoff in this game has the form $H(x, y) = EH_z(x, y)$, where

$$H_z(x, y) = \begin{cases} \frac{x+y}{2}, & \text{if } x \leq y \\ x, & \text{if } x > y, |x-z| < |y-z|, \\ y, & \text{if } x > y, |x-z| > |y-z|, \\ z, & \text{if } x > y, |x-z| = |y-z|. \end{cases} \quad (1)$$

In the papers [2]-[4] the equilibriums in the final-offer arbitration procedure in mixed strategies were found.

Let $-\infty < y \leq 0 \leq x < +\infty$ and $\alpha > 0$. Suppose

$$H_z(x, y) = \begin{cases} x^\alpha, & \text{if } |x-z| < |y-z|, \\ -(-y)^\alpha, & \text{if } |x-z| > |y-z|, \\ z, & \text{if } |x-z| = |y-z|. \end{cases} \quad (2)$$

Let the arbitrator chooses one of the $2n+1$ numbers: $-n, -(n-1), \dots, -1, 0, 1, \dots, (n-1), n$ - with equal probabilities $p = \frac{1}{2n+1}$.

We find the equilibrium in the game in mixed strategies. Denote the mixed strategies of the players L and M by $f(x)$ and $g(y)$, respectively. We have:

$$f(x) \geq 0, \int_0^{+\infty} f(x)dx = 1; g(y) \geq 0, \int_{-\infty}^0 g(y)dy = 1.$$

Due to the symmetry, the game price is equal to zero, and the optimal strategies are symmetric with respect to the y -axis, i.e. $g(y) = f(-y)$. Hence, it suffices to construct the optimal strategy only for one player, for example, L .

Theorem 1. If $\alpha \in (0, 2]$ and $n = 1$, then for the player L the optimal strategy is

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x < c, \\ \frac{\alpha c^{\frac{\alpha}{2}}}{x^{\frac{\alpha}{2}+1}}, & \text{if } c \leq x < c+2, \\ 0, & \text{if } c+2 \leq x < +\infty \end{cases} \quad (3)$$

where $c = \frac{2}{4^{\frac{1}{\alpha}} - 1}$

Theorem 2. If $\alpha = 1$, then for the player L the optimal strategy is

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x < c, \\ \frac{(n+1)\sqrt{c}}{2\sqrt{x^3}}, & \text{if } c \leq x < c+2, \\ 0, & \text{if } c+2 \leq x < +\infty \end{cases} \quad (4)$$

where $c = \frac{2n^2}{2n+1}$

Theorem 3. If $\alpha=2$, then for the player L the optimal strategy is

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x < c, \\ \frac{(n+1)c}{x^2}, & \text{if } c < x < c+2, \\ 0, & \text{if } c+2 < x < +\infty \end{cases} \quad (5)$$

where $c = 2n$

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Periodicals in Game Theory

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On Bilateral Barters and Market Equilibrium

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Keywords: *Exchange economies, Market equilibrium, Transferable utility, Common price*

We study an economy in which a fixed finite set I of economic agents exchange commodity bundles or contingent claims, in order to maximize their own utilities. Important instances which are relevant to management of energy and resources comprise exchange of natural resources or user-rights to such, transfers of fish quotas, production allowances, pollution permits, or rights to water use. The exchange of contingent claims, in which case, what comes into focus is mutual insurance or security exchange, is also an important economy of that type.

In that setting, it is well known that when transfers are costless and multilateral trades can be arranged at no expense, and when some technical conditions are fulfilled, voluntary trade will lead inevitably to Pareto optimality. However, when transfers are costly and multilateral trades difficult or impossible, optimality is problematical. In that case, this paper asks: Can equilibrium - whence Pareto efficiency - obtain merely via repeated, bilateral exchange? (several studies consider links between pairwise, t -wise, and overall Pareto optimality; see e.g. Feldman (1973), Madden (1975), Goldman and Starr (1982), Fisher (1989)). And, most importantly: may traders dispense with optimization? Moreover, while exchange still remains in swing, can parties proceed without announcement of prices? That is, might market equilibrium - and the attending

prices - emerge as final outcomes, identifiable only after all desirable transactions are completed?

In short, the question addressed is whether pair-wise, direct deals can lead market agents to coordinate on equilibrium prices? The approach fits adaptive learning, behavioral economics, and decentralized coordination. Focus is on feasible, voluntary exchanges, driven only by differences in substitution rates. Presuming transferable utility, we provide sufficient conditions for convergence to market equilibrium. It facilitates commodity transfers and price coordination that some parties have differentiable objectives or make strictly feasible choices. In quite common, most convenient cases each constraint set is polyhedral.

To be specific, first we define the setting and the concept of equilibrium solution. In order to approach such outcomes, we elaborate on bilateral barterers. Then, we propose a repeated bilateral barterers process and study complete trade and convergence. Finally, we analyze the attainment of equilibrium.



ВЕСТНИК САНКТ-ПЕТЕРБУРГСКОГО УНИВЕРСИТЕТА

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Solidary solutions to games with restricted cooperation

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Keywords: *Cooperative games, Claim problem, Proportional method, Proportional nucleolus, Nucleolus, Least square solution.*

A *TU-cooperative game with restricted cooperation* is a quadruple (N, A, c, v) , where N is a finite set of agents, A is a collection of nonempty coalitions of agents, c is a positive real number (the amount of resources to be divided by agents), $v = \{v(T)\}_{T \in A}$, where $v(T) > 0$ is a claim of coalition T . We assume that A covers N .

A *set of imputations* of (N, A, c, v) is the set $\{y_i\}_{i \in N} : y_i \geq 0, \sum_{i \in N} y_i = c\}$. A *method* F is a map that associates to any game (N, A, c, v) a subset of its set of imputations. Then $F(N, A, c, v)$ is the *solution* of (N, A, c, v) . We denote $y(S) = \sum_{i \in S} y_i$.

If $A = \{i\} : i \in N$ then a *claim problem* arises. Claim methods and their axiomatic justifications are described in the survey [1]. For almost all solutions of claim problems, if one agent gets less than its claim then all agents get less than their claims, i.e., the *solidarity property* takes place.

In this paper we consider generalizations of the solidarity property to games with restricted cooperation. We obtain conditions on A that ensure existence of generalized solidary solutions and conditions on A that ensure solidarity properties for generalizations of the Proportional method and of the Uniform Losses method for claim problems.

Let $X \subset R^n$, u_1, \dots, u_k be functions defined on X . For $z \in X$, let π be a permutation of $\{1, \dots, k\}$ such that $f_{\pi(i)}(z) \leq f_{\pi(i+1)}(z)$, $\theta(z) = \{f_{\pi(i)}(z)\}_{i=1}^k$. Then $y \in X$

belongs to the *nucleolus with respect to* u_1, \dots, u_k on X iff $\theta(y) \geq_{lex} \theta(z)$ for all $z \in X$.

We consider the following generalized methods.

1. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Solidary solution* of (N, A, c, v) ($y \in SS(N, A, c, v)$) iff $x(Q) < v(Q)$ for some $Q \in A$ implies $x(T) < v(T)$ for all $T \in A$.

2. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Weakly Solidary solution* of (N, A, c, v) ($y \in WSS(N, A, c, v)$) iff $x(Q) < v(Q)$ for some $Q \in A$ implies $x(T) < v(T)$ for all $T \in A$ with $Q \cap T = \emptyset$.

3. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Proportional solution* of (N, A, c, v) ($y \in P(N, A, c, v)$) iff $y(S)/v(S) = y(Q)/v(Q)$ for all $S, Q \in A$.

4. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Weakly Proportional solution* of (N, A, c, v) ($y \in WP(N, A, c, v)$) iff $y(S)/v(S) = y(Q)/v(Q)$ for all $S, Q \in A$ such that $S \cap Q = \emptyset$.

5. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Uniform Losses solution* of (N, A, c, v) ($y \in UL(N, A, c, v)$) iff for all $S, T \in A$, $x(S) - v(S) < x(T) - v(T)$ implies $x(T) = 0$.

6. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Weakly Uniform Losses solution* of (N, A, c, v) ($y \in WUL(N, A, c, v)$) iff for all $S, T \in A$, $x(S) - v(S) < x(T) - v(T)$ and $S \cap T = \emptyset$ imply $x(T) = 0$.

7. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Proportional Nucleolus* of (N, A, c, v) ($y \in PN(N, A, c, v)$) iff y belongs to the nucleolus w.r.t. $\{u_T\}_{T \in A}$ with $u_T(z) = z(T)/v(T)$ on the set of imputations of (N, A, c, v) .

8. An imputation $y = \{y_i\}_{i \in N}$ belongs to the *Nucleolus* of (N, A, c, v) ($y \in N(N, A, c, v)$) iff y belongs to the nucleolus w.r.t. $\{u_T\}_{T \in A}$ with $u_T(z) = z(T) - v(T)$ on the set of imputations of (N, A, c, v) .

9. Let G be a class of strictly increasing continuous functions g defined on $(0, +\infty)$ such that $g(1) = 0$ and $\lim_{x \rightarrow 0} \int_a^x g(t) dt < +\infty$ for each $a > 0$.

A vector $y = \{y_i\}_{i \in N}$ belongs to g --solution of (N, A, c, v) ($y \in gS(N, A, c, v)$)

iff y minimizes $f(z) = \sum_{S \in A} \left(\int_{v(S)}^{z(S)} g(t/v(S)) dt \right)$ on the set of imputations of (N, A, c, v) .

For $A = 2^{[N]} \setminus \{N, \emptyset\}$, g --solutions were defined in [3].

10. A vector $y = \{y_i\}_{i \in N}$ belongs to the *Least Square solution* of (N, A, c, v) ($y \in LS(N, A, c, v)$) iff y minimizes $\sum_{S \in A} (v(S) - z(S))^2$ on the set of imputations of (N, A, c, v) .

For each A , $c > 0$, v with $v(T) > 0$, the Proportional Nucleolus, the Nucleolus, the g --solution, and the Least Square solution of (N, A, c, v) are nonempty sets and define uniquely total amounts $y(T)$ for each $T \in A$.

The conditions on A that ensure existence of the Solidary solution coincide with conditions on A for existence the Proportional solution and coincide with conditions on A for existence the Uniform Losses solution: A must be a minimal covering of N .

The conditions on A that ensure existence of the Weakly Solidary solution coincide with conditions on A for existence the Weakly Proportional solution and coincide with conditions on A for existence the Weakly Uniform Losses solution, described in [2], Theorem 3.

The following theorems describe solidarity properties of the generalized solutions.

Theorem 1. *For each $g \in G$, the following statements are equivalent.*

$SS(N, A, c, v) \supset PN(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$P(N, A, c, v) \supset PN(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$SS(N, A, c, v) \supset N(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$UL(N, A, c, v) \supset N(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$SS(N, A, c, v) \supset gS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$P(N, A, c, v) \supset gS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$SS(N, A, c, v) \supset LS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$UL(N, A, c, v) \supset LS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

A is a partition of N .

Theorem 2. *The following statements are equivalent.*

$WSS(N, A, c, v) \supset PN(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$WP(N, A, c, v) \supset PN(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$WSS(N, A, c, v) \supset N(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$WUL(N, A, c, v) \supset N(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$.

The condition on A that is equivalent to these statements is described in [3], Theorem 2.

Theorem 3. *For each $g \in G$, the following statements are equivalent.*

$WSS(N, A, c, v) \supset gS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$WP(N, A, c, v) \supset gS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$WSS(N, A, c, v) \supset LS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$WUL(N, A, c, v) \supset LS(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$.

The condition on A that is equivalent to these statements is described in [3], Theorem 1.

Theorem 4. *For each $g \in G$, the following statements are equivalent.*

$gS(N, A, c, v) = WP(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$PN(N, A, c, v) = WP(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$;

$N(N, A, c, v) = WUL(N, A, c, v)$ for all $c > 0$, all v with $v(T) > 0$.

The condition on A that is equivalent to these statements is described in [3], Theorem 3.

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Optimal Investment Strategy to Minimize the Ruin Probability of an Insurance Company

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Keywords: *Insurance, Classical risk model, Ruin probability, Cramér–Lundberg process, Optimal investment strategy*

A generalization of collective risk model is considered. We propose a model in view of the possibility of investing part of the surplus in a risky asset. The assumption allows minimizing the probability of ruin of an insurance company by investment. This generalization of classical model was computer simulated. The exact solutions and asymptotic approximations to the probability of ruin were obtained.

Game-theoretic Analysis of Strategic Export Pricing in the Presence of Anti-dumping Practices

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Keywords: *Credibility Dynamic game, Pricing, Anti-dumping*

The term “dumping” refers to the exporting firms’ practice of charging a lower price in the export market than in the domestic market. Many countries utilize anti-dumping (AD) duties to protect domestic producers from unfair import competition. Anti-dumping duties are gradually becoming the single most prominent trade policy instrument. They are under direct control of policy makers, usually large in size, and produce strong welfare effects. A number of strategic considerations shape exporting firms’ pricing decisions as well as actions by the importing country trade authorities. The proposed research aims to disentangle numerous factors affecting those decisions and may help devise more optimal AD policies.

A large part of prior theoretical research on anti-dumping recognizes the importance of strategic considerations for decisions of participants in the AD process and therefore utilizes elements of game theory. Early work in that area (Fischer 1992, Reitzes 1993, Blonigen and Prusa 2003, and others) focused on the preventive effect of AD laws. Those papers show how the mere presence of an AD law in a country induces exporters to that country to reduce the volume and raise the price of exports to avoid AD suits and charges. Anderson (1992, 1993) uses the notion of strategic behavior to arrive at the opposite result. He demonstrates that under some conditions the presence of an AD law induces exporting firms to increase the volume of exports. Therefore, the adoption of an AD law may in fact increase the extent of dumping. Vanderbussche and Zanardi (2008) also contributes to the literature by arguing that the proliferation of AD laws is largely explained by retaliatory motives.

Many of those theoretical findings have also been supported by empirical research (Rutkowski 2007, Blonigen and Bown 2003, Blonigen and Park 2004, and others).

Nizovtsev and Skiba (2010) examined an exporter's entry into a foreign market along with a subsequent AD investigation, presenting those processes as a sequential game. An exporting firm's anticipation of the second-stage outcome is incorporated in the first-period decisions. The paper predicts that the exporting firm is more likely to raise its price in response to the AD duty if import demand is less elastic. The analysis in the paper focuses on the last stage of the pricing game, namely the one that follows the imposition of the AD duty. There are, however, intriguing strategic considerations that precede that stage. Earlier decisions affect later stages and the outcomes of the anti-dumping proceedings. Up to now, the existing theoretical research on anti-dumping has been silent on the issue of how the exporter chooses its entry price for the foreign market. Due to the extremely large number and complexity of factors involved in exporters' decisions, their actions often remain a mystery. As Blonigen and Prusa (2003) concluded in their comprehensive review of work on AD up to that point, "Perhaps frustratingly, ...this can lead to just about any combination of distorted market effects, depending on the characteristics of the strategic game being played by the firms."

Our present research carries out a further game-theoretic inquiry into the behavior of exporting firms in the presence of AD legislation in the recipient country. We endogenize the firm's entry price decision and explore the pricing trajectory across time in a dynamic incomplete information game setting. Doing so allows us to disentangle various factors that affect the pricing decision and place this research in the general context of industrial organization research on pricing.

We examine how the optimal pricing path and therefore the optimal anti-dumping policy are affected by the following factors:

- The country's demand for imports can stay constant, increase over time due to consumer learning, or decrease due to obsolescence of products with a short lifecycle.
- The structure of the distribution chain in the importing country. If the price for the domestic market is set by an independent profit-maximizing intermediary, that firm may play a dynamic game with the exporting firm. That fact changes the equilibrium outcomes and has implications for trade policies.
- The specifics of the AD duty collection procedures. The United States collects AD duties retroactively – the size of a duty in a given year is based on the exporter's pricing in the previous year. In this aspect, the U.S. procedure is strikingly different from that used in

most other countries including EU, which use a ‘pay-as-you-go’ scheme. Intuitively, a retroactive duty is more likely to lock an exporter out of the market completely, but full comparative welfare analysis of the two schemes hasn’t been done yet.

Our work makes an important step towards disentangling the numerous complex considerations involved in AD policy by investigating previously unexplored facets of anti-dumping. Better understanding of strategic motives behind exporters’ price-setting and their likely responses to AD duties may help improve the accuracy of AD policies conducted by importing countries, which would in turn have a positive effect on their welfare. As Gallaway et al. (1999) demonstrate, the welfare effect of AD duties can be quite substantial, more so when exporters respond by raising their prices. Our research may also have broader implications by providing insight useful for crafting trade policies other than AD. At the very least, there seems to be a case for including expectations of exporters’ responses into all trade policy decisions in accordance with the principles of game theory. That would increase the chances that trade policies will in fact achieve their goals.

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Sequential Bargaining Scheme in the Assets Sharing Problem

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Keywords: *Markov strategy with memory, Resource sharing, Priority, Discounting.*

We consider a model of the sequential game related with resource sharing. Such situation arises, for example, when the owners of any two companies decide to join, merging their assets and making preliminary agreement on the distribution of total assets. Each of them, with a certain probability, can remove a partner from the management and thus obtain all of the assets. When the agreement is concluded, each player must decide whether he will try to eliminate the opponent or not. In discrete periods the decision is made again. Moreover, at each stage the payoff is discounted. It is assumed, that players use Markov strategies with memory of one or two units and horizon of negotiations is infinite.

The games with a priority and without for the fixed probabilities of attempt to eliminate the opponent, are investigated and equilibrium in pure strategies is found.

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On an algorithm for determination of Nash equilibria in the informational extended 2-matrix games

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Keywords: *Informational extended game, Nash equilibria, bimatrix game, extended matrices*

The player's possession of supplementary information about unfolding of the game can influence appreciably the player's payoffs. Thus for the same sets of strategies and the same payoff functions it is possible to obtain different results, if the players have supplementary information.

Let us consider the 2-matrix game in the normal form $\Gamma = \langle N, X_1, X_2, A, B \rangle$, where $A = \{a_{ij}\}$, $B = \{b_{ij}\}$, $i = \overline{1, m}$, $j = \overline{1, n}$ (A and B are the payoff matrices for the first and the second player respectively).

There are 2-matrix games for which the set of the Nash equilibria is empty: $NE(\Gamma) = \emptyset$ (solutions do not exist in pure strategies). For every 2-matrix game we can construct some informational extended games, for which always there exists at least one solutions in pure strategies (Nash equilibria) (see [2],[3]).

According to [1], let us define two forms of informational extended games: ${}_1\Gamma$ and ${}_2\Gamma$. We consider that for the game ${}_1\Gamma$ the first player knows the chosen strategy of the second player, and for the game ${}_2\Gamma$ the second player knows the chosen strategy of the first player. If one of the players knows the chosen strategy of the other, then the set of the strategies for this player can be represented by a set of mappings defined on the set of strategies of his opponent.

Definition 1. (The game ${}_1\Gamma$, [1]) The informational extended 2-matrix game ${}_1\Gamma$ can be defined in the normal form by: ${}_1\Gamma = \langle N, \overline{X}_1, X_2, \overline{A}, \overline{B} \rangle$, where $N = \{1, 2\}$, $\overline{X}_1 = \{\varphi_1 : X_2 \rightarrow X_1\}$, $\overline{A} = \{\overline{a}_{ij}\}$, $\overline{B} = \{\overline{b}_{ij}\}$, $i = \overline{1, m^n}$, $j = \overline{1, n}$.

For the game ${}_1\Gamma$ we have $\overline{X}_1 = \{1, 2, \dots, m^n\}$, $X_2 = \{1, 2, \dots, n\}$, $|\overline{X}_1| = m^n$, and the matrices \overline{A} and \overline{B} have dimension $[m^n \times n]$ and they contains the elements of initial matrices A and B respectively.

The matrices \overline{A} and \overline{B} will be constructed in the next mode. Let us denote by A_i , B_i , $i = \overline{1, m}$ the rows i in the matrices A and B , respectively). Choosing one element from each of these rows A_1, A_2, \dots, A_m , we will build one column in the matrix \overline{A} . The columns from the matrix \overline{B} are built in the same mode, choosing one element from each of the rows B_1, B_2, \dots, B_m .

Similarly, we can define the informational extended game ${}_2\Gamma$. For the generation of the extended matrices \overline{A} and \overline{B} we can use the next methods.

The first method is based on representation of decimal numbers in the base which represent the number of rows or the number of columns in the initial matrices.

For the game ${}_1\Gamma$ we need to represent the numbers $0, 1, \dots, (m^n - 1)$ in the base m with n components: $N_m = (C_0 C_1 \dots C_{n-1})_m$, where $C_j \in \{0, 1, \dots, m-1\}$, $j = \overline{0, n-1}$, that is $(C_0 m^0 + C_1 m^1 + \dots + C_{n-1} m^{n-1}) = N_{10}$. Each of these numbers N_m represented in the base m will correspond to one column in the extended matrix.

Then for elements from column j it must replace as it follows:

$$0 \rightarrow a_{1j}, 1 \rightarrow a_{2j}, \dots, i \rightarrow a_{(i+1)j}, \dots, (m-1) \rightarrow a_{mj} \text{ (similarly for the } B \text{)}.$$

The second method consists in assigning two numbers to each of the elements from the initial matrices. One of these numbers represents the number of blocks (series) formed by this element, and the second number represents the length of the block (that is, the number of repetitions of this element in the block).

Denote by nrb_l the number of blocks for some element a_{ij} (b_{ij}) and by L the length of each of blocks (the number of repetitions of this element into the block).

So for the game ${}_1\Gamma$ we assign to each element from the column $j: (m^{j-1})$ blocks (series) each of them with the length (m^{n-j}) . Thus for all elements $\forall i = \overline{1, m}, j = \overline{1, n}$, we determine the indices of the rows k of this element in the extended matrix.

In such mode for the element from the row i and from the column j and for all $nrb l = \overline{1, m^{j-1}}, L = \overline{1, m^{n-j}}$ we calculate the number k by:

$$k = m \cdot m^{n-j} \cdot (nrb l - 1) + (i - 1) \cdot m^{n-j} + L.$$

Thus, we construct the matrices \overline{A} and \overline{B} : $\overline{A}[k, j] = A[i, j]$, $\overline{B}[k, j] = B[i, j]$.

Remark. These two methods may be used independently. Using it we can construct the extended matrices entirely or partly. If the initial matrices are very big, we can use these methods for partial construction of the extended matrices. Thus the first method may be used when we need to construct only one row (or only one column) for the game ${}_1\Gamma$ (${}_2\Gamma$), and the second method may be used when we need to determine the position of some element in the extended matrix, i.e. the index of the row (or of the column) in the game ${}_1\Gamma$ (or ${}_2\Gamma$, respectively).

Using these methods we can construct an algorithm for determination of Nash equilibria in the informational extended 2-matrix games, which not need the integral construction of the extended matrices. Thus we avoid using a big volume of memory, since the extended matrices will have a big dimension ($[m \times n^m]$).

Algorithm. Consider the extended game ${}_1\Gamma$.

Using the first method we construct each row in the extended matrix \overline{B} .

For each row i_0 ($i_0 = \overline{1, m^n}$) from the matrix \overline{B} we will do the next operations.

1. We determine the maximum element from this row of the extended matrix \overline{B} , and the corresponding element with the same indices from the matrix \overline{A} ; let them be $\overline{b}_{i_0 j_0}$ and $\overline{a}_{i_0 j_0}$.

2. We determine the maximum element from the column j_0 in the initial matrix A : let's consider this element $a_{i^* j_0}$.

3. If $\bar{a}_{i_0 j_0} = a_{i^* j_0}$, then (i_0, j_0) is *NE* equilibrium for the extended game ${}_1\Gamma$: $(i_0, j_0) \in NE({}_1\Gamma)$, and the elements $\bar{a}_{i_0 j_0}$ and $\bar{b}_{i_0 j_0}$ will be the payoff's values for the first and for the second player respectively.

Remark. In the case when the numbers n^m and m^n are very big this algorithm for determination of *NE* equilibria for the informational extended games and the generation methods of the extended matrices are more complex. But all these operations can be executed operating with the corresponding numbers represented in the base m or n respectively to the informational extended games $({}_1\Gamma$ or ${}_2\Gamma)$.

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МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

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The Emergence of Crises and the Role of Heterogeneous Expectations

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Keywords: *Differential games, Incentive equilibrium strategies, Credibility*

In the literature several approaches have been proposed to ensure the sustainability over time of an agreement reached at the starting date of a differential game. One of the approaches appropriate for two-player differential games is to support the cooperative solution by incentive strategies, [2], [3]. Incentive strategies are functions of the possible deviation of the other player and recommend to each player to implement his part of the agreement whenever the other player is doing so. The equilibrium incentive approach allows embodying the cooperative solution with an equilibrium property. Therefore, by definition each player will find individually rational to stick to his part in the coordinated solution. One important property that should be checked is the credibility of these incentive strategies, [3], [4]. These strategies are credible if each player will implement his incentive strategy and not the coordinated solution if he observes that the other one has deviated from the agreement. Recently some papers, [5], [6], have provided conditions to check for the credibility of incentive strategies for the class of linear-state and linear-quadratic differential games. To preserve the special structures of the games the analyses have been restricted to linear incentive strategies that are not always credible.

The recent financial crisis and the subsequent global economic downturn has revived the debate about the foundations of standard macroeconomic models, and their usefulness in explaining real-world developments as the one we have just experienced. In particular, the realism of the idea that agents of the economy are homogenous and behave rationally has been questioned as unrealistic and unable to explain the strong

periodical macroeconomic fluctuations and the periodical crises.

In this paper, we try and explain the reasons of the emergence of occasional strong macroeconomic fluctuations by assuming that the economic agents can form heterogeneous expectations according to different models. In order to avoid falling into arbitrariness in modeling expectations, we set our population of models in the context of a genetic algorithm, which limits the space of possible agent models by selecting away those which do not perform, as market forces would do. The possible agents' models include RE as part of a wider set of possible expectations. We proceed as follows. First, we motivate our choice of a GA by comparing it with other possible relaxation of RE. We describe the GA as a technique of generalizing the strict RE paradigm while retaining the attractive feature that the agents are able to optimize given their beliefs, and argue that a GA has some advantages over other similar techniques because it is less demanding in terms of hypotheses, it fits more data (in particular, expectations) and more accurately, it can be easily mapped into activities observed at the micro level, and it is microfounded in a loose sense.

In the second part of the paper, after having introduced and justified the use of the GA, we describe its actual functioning in the context of a simple macro model and propose our results.

The notable results are the following: persistence in the data series and “fat tails” are generated endogenously, without recurring to autocorrelated disturbances; heterogeneous expectations are generated and compared with existing expectations surveys; finally, and most importantly, we are able to explain under which conditions occasional strong deviations from equilibrium may arise and “surprise” the markets and the forecasters. The use of a GA also provides with interesting insights when the models are subject to structural changes. These will be the topic of further research

The genetic algorithm

The modelling device we choose is a Genetic Algorithm (henceforth GA). A genetic algorithm is (along with evolution strategy and genetic programming) a class of stochastic search algorithms based on analogies with biological evolution.

As a relaxation of RE, a GA model has merits which are similar to those of learning: it eliminates some very restrictive hypotheses of RE. The difference (and possibly the advantage of this modeling solution) is that the hypothesis of knowledge of the model can be not only relaxed but even completely removed, as agents may be

assumed to ignore the structure of the model. In a GA model, even in those extreme versions where the agents do not know the correct model of the economy, they are able to use the GA as a heuristic to gather information. An extensive comparison of Genetic Algorithms with other forms of learning can be found in Brenner (2006) and Duffy (2006).

The hypothesis of rationality is maintained, in the sense that agents are still able to optimize on the basis of their beliefs. The main disciplinary device is kept, in that the actions of the agents in the model are optimal given their beliefs.

Given that in a GA the agents have almost by definition heterogeneous beliefs, the hypothesis of coordination of the expectations is also not necessary, as the representative agent is abandoned. This generalization comes with a price: GA models cannot be analytically solved but only simulated. We do not believe this is a real problem for several reasons: first, most RE models (with the exception of the most simple ones) are also solved using simulation techniques; second, modern computers have made these techniques inexpensive; third and most important, the GA is a well known technique used in several fields such as biology and engineering which relies on a solid theoretical background and provides reliable results.

This model departs from the prevailing literature in three dimensions. First, it produces endogenously varying shares of both “rational” and “non-rational” (heterogeneous) agents. As a matter of fact, in our model a high and varying heterogeneity of agents is not only permitted, but is an essential ingredient. It also leads to empirically testable implications about first and second moments of expectations.

Second, the evolution of the model based on a GA describes in a stylized way well-known micro phenomena of learning such as survival of the best ideas over time, modification of existing ones, experimentation, and imitation.

Third, we emphasize the difference between genotype (the model used internally by the agent) and phenotype (the forecast apparent to the others). Our expectations are not derived from a population of simple rules (e.g. as in Lettau (1997) or in LeBaron (2006)), but stem from models of the economy. Additionally, agents can get closer to the right forecast either by using RE or by imitating, thus making the same prediction in equilibrium and obtaining the same payoff, but in fact using completely different models.

What are the modelling advantages of a GA? Many advantages are in common with learning. Both GA and learning produce persistence in variables without recourse to exogenous hypotheses (such as autocorrelated shocks). GA operators (imitation, mating etc) correspond to known heuristics described in micro theory just like learning corresponds to known ways of getting to know the parameter of the data. Both approaches are less restrictive than RE because they do not suppose that the agent knows more than the modeller, but GA can go as far as allowing the agents to ignore important features (such as the structure) of the model. Quite importantly to explain periods of turbulence, GAs are able to produce endogenous fluctuations beyond those imposed with the shocks, propagating them not only in time but also increasing their size. A final important advantage of GAs is that they provide a technology which makes it easier and almost natural to model heterogeneous agents. Thus, uncertainty as disagreement and biases in expectations can be part of the model. This is a relevant aspect in empirical assessment of a model, since expectations and macroeconomic variables are modelled together and the expectations produced by the model can be compared with those measured from surveys

The model

We use the simplest possible framework, an economy expressed by one equation only (a “Phillips curve”):

$$p(t)a + b * E(p(t)) + u(t)$$

We therefore abstract from any pretension of realism, in favour of a model in which the outcomes can be clearly attributed to the action of the agents rather than to the equations of the model. In doing this, we follow Sargent (1999), who uses as a workhorse a version of Bray's (1982) pricing model. As in Sargent, we emphasize a relevant characteristic that will be the object of our study, namely the forward-looking term in the equation. We assume that prices are a function of the aggregate expectations, as in Kirman (1991) and De Grauwe (1993). The true model (the actual law of motion) varies depends on how the agents' model is specified. We also allow for backward looking expectations in some versions of our model.

The expectations of the agents evolve within the GA. Within the GA, agents can also choose to be true

RE-believers, as for example in Baron (2006). The RE-believers are given the correct model of the economy and immediately reach the RE equilibrium. However,

and differently from Baron, when other types of agents are introduced, RE-believers erroneously continue to assume that all the other agents will behave as they do¹, while this is not true because agents who do not know the true model of the economy search for the best possible prediction by using heuristic methods.

Each agent can also choose to use another model produced in the GA. Reasons to use a model other than RE include 1) ignorance of the exact parameters of the model, 2) lack of knowledge about the preferences of the other agents, or 3) awareness that other agents have heterogeneous preferences. If either of these conditions is met, the agent (knows that he) does not know the true model of the economy; furthermore, those who believe that expectations play some role in determining outcomes must also be aware that the optimum is a shifting one.

For analytical purposes, while running our GA, at each point in time we keep track of three ‘types’ of agents: 1) those choosing rational expectations, 2) those using other models, and within the latter 3) those ending up (via imitation and mutation) having a model to form expectations which is formally identical as the RE. We call the second group non-RE and the last group RE-imitators. It turns out that the RE-imitators are essential in speeding up the process of equilibrium when they show a ‘herd behaviour’ towards RE, but may also be responsible of large and protracted deviations from it.

Results

One interesting feature of the way agents interact is that they are able to generate endogenous cycles and fat tails. What happens indeed is that, when rational and pseudo-rational agents dominate the population, there is no longer any comparative advantage in being orthodoxically rational, as the equilibrium is not affected. Therefore, rational agents start to decrease, and the presence of more irrational agents moves the economy away from the equilibrium. When the shift is significant, it becomes again convenient to be rational, and rational agents again prevail in a way similar to the one described in LeBaron (2006)

¹ The alternative hypothesis would be a very demanding one, as discussed in the previous

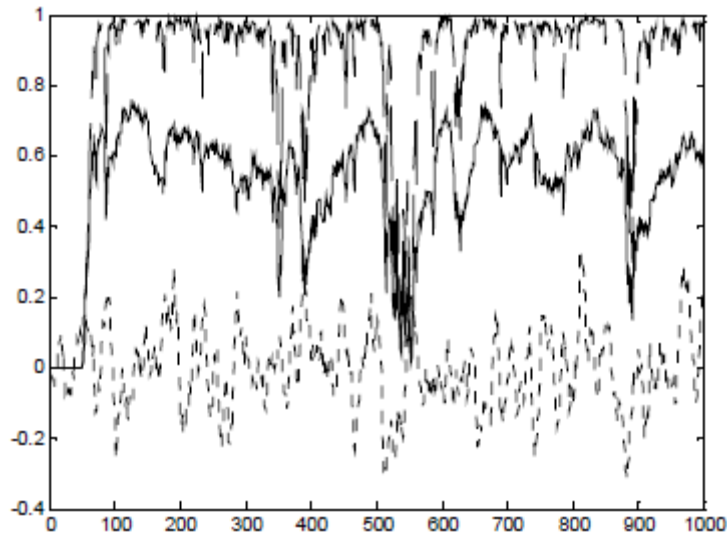


Figure 1. Example of endogenous cycle: macro outcomes (dots), percentage of rational agents (solid line), percentage of agents either rational or imitating rational agents (dotted line).

Corresponding to our expectations, the (unique) RE equilibrium emerges for most parameterizations of the model, but the convergence takes more time in forward-looking models. We observe that in most cases² rational agents have a higher fitness when the system is out of equilibrium; therefore other agents either switch to true RE or they simply adjust the parameters in their models to produce expectations which are close to the RE.

We also observe that, once attained, the RE equilibrium can be sustained for protracted periods despite the presence of a small number of rational agents. To understand this point, it is important to note that when the RE equilibrium is attained, all non rational agents have also RE-compatible expectations, despite not using the RE model. Therefore, as long as the effect of mutation is negligible, the choices of RE and boundedly rational agents are observationally equivalent: RE agents receive the same fitness than those who are just imitating their choices and their number in the total population becomes a random walk. The RE equilibrium may hold for protracted

²Sargent (1999) shows that if agents behave like econometricians and use a fixed gain algorithm the RE equilibrium is substituted by a stationary stochastic process. The true model (the actual law of motion) varies depending on how the agents' model is misspecified. Our model reproduces this result, but due to the presence of heterogeneous expectations we show that these equilibria are temporary and relatively infrequent.

periods even when all the RE agents have disappeared.

However, over time the effects of mutation become important and move the equilibrium away. Depending on the parameterization of the model (more precisely on the degree of forward-lookingness of the model) and on the degree of coordination of the expectations of the non rational agents, the system can drift away for quite prolonged amounts of time. These periodical deviations cannot be permanently sustained, as the potential advantage of RE agents in term of fitness increases with the distance of the system from the equilibrium.

Following a structural break, the RE agents instantaneously adjust, while the boundedly rational agents cannot do it. The new dynamic to equilibrium sees an increasing number of agents either adopting RE or imitating them. The variance in the expectations increases in the transition phase, in accordance with the empirical observation.

Looking at the data generated, a GA model can induce not only endogenous persistence in the series but also multiply an initial shock and create endogenous cycles following a small disturbance. It also implies, for plausible parametrizations, multiple equilibria following a shock, and the selected equilibrium does not depend on the disturbance itself but on the characteristics of the population of models at the time. The observed paths remind of sunspot equilibria.

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Cooperative Agreements in ATM Network Management

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Keywords: *ATM network management, Network formation, Coalition, Allocation.*

We consider the situation which can take place on the market of several firms which are the providers of the same services to the inhabitants of some region. These firms have to incur similar expenses to provide access to the usage of the services which are rendered by the firms. Each firm has a quantity of units of the so-called resource, e. g. if we consider a bank as a firm, the bank, in order to serve its customers, should set up in the region where it works ATMs which are the resource of the bank. When banks consolidate in a network the pooling of the ATMs takes place. It allows banks to improve the quality of the service of their clients and at the same time to save money on the sharing of additional units of the resource. In this case the expenses allocation problem among the banks which have pooled their ATMs in a network arises. The similar problem and its solution is considered in the works [1, 3, 4, 7].

Suppose we have a finite set of banks N in a region L . The region consists of a finite number of locations $\ell \in L$, in which banks may set up its ATMs. It is also supposed that number of clients of each bank in each location can be accurately estimated and commonly known. A client whether can use ATMs of any bank for transactions in the location or use non-ATM method of getting money, but it reflects on client's bank transaction costs. Assume that bank $i \in N$ has two numerical characteristics for each location: n_i^ℓ is the number of transactions of bank i in location $\ell \in L$ and k_i^ℓ is the number of ATMs owned to bank i in location ℓ . Let $S \subseteq N$ and $n^\ell(S) = \sum_{i \in S} n_i^\ell$ be equal to the number of transactions of the banks from S in location ℓ and $k^\ell(S) = \sum_{i \in S} k_i^\ell$ be equal to the number of ATMs of the banks from S in location

ℓ . If the client of bank i uses ATM of bank i the expenses of bank i are equal to $\alpha > 0$. If the client of bank i uses ATM of bank j which has a bilateral agreement with bank i the expenses of bank i are equal to $\beta > \alpha$. If the client of bank i uses ATM of bank j which has not a bilateral agreement with bank i or uses non-ATM method of getting money the expenses of bank i are equal to $\gamma > \beta$.

We make two suggestions about behavior of the clients in the location:

- The client of bank i uses ATM for his transaction in the location ℓ if bank i or/and the banks which have bilateral agreements with bank i have ATMs in location ℓ
- The client of bank i uses ATM of bank i and ATM of bank j which has a bilateral agreement with bank i for his transaction with equal probabilities.

Let A^ℓ be a set of the banks that have ATMs in location ℓ . Suppose that players from $S \subseteq N$ form a network of ATMs. For any location $\ell \in L$ the total transaction costs of coalition S equal to the following ones:

$$c^l(S) = \begin{cases} \alpha \sum_{i \in S} \frac{k_i^\ell}{k^\ell(S)} n_i^\ell + \beta \sum_{i \in S} \left(1 - \frac{k_i^\ell}{k^\ell(S)}\right) n_i^\ell, & \text{if } S \cap A^l \neq \emptyset \\ \gamma n_i(S), & \text{if } S \cap A^l = \emptyset \end{cases},$$

We can consider a cost savings game for location ℓ with characteristic function v^ℓ determined by the following way:

$$v^\ell(S) = \sum_{i \in S} c^\ell(\{i\}) - c^\ell(S) = \begin{cases} (\gamma - \beta) \sum_{i \in S \setminus A^\ell} n_i^\ell - (\beta - \alpha) \sum_{i \in S \cap A^\ell} \left(1 - \frac{k_i^\ell}{k^\ell(S)}\right) n_i^\ell, & \text{if } S \cap A^l \neq \emptyset \\ 0, & \text{if } S \cap A^l = \emptyset \end{cases}$$

If there are more than one location, we can find the characteristic function of the game like this:

$$v(S) = \sum_{\ell \in L} v^\ell(S).$$

Using the function $v(S), S \subseteq N$ we propose a game of agreement formation [2,5,8] for reducing bank costs and allocation rules [9].

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Periodicals in Game Theory

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The Shapley Value, the Convex Games and the Steiner Point of the Convex Sets

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Keyword: *The Shapley value, Convex games, The Steiner point, The differences of the cores.*

A new interpretation of the Shapley value using the Steiner point of the core is considered.

Let $N=\{1,2,\dots,n\}$ be a set of players, and v be an arbitrary TU game. The core of a game v is denoted by $C(v)$.

A game v is convex if it satisfies the inequalities $v(S)+v(T)\leq v(S\cap T)+v(S\cup T)$ for all coalitions S and T . The core of a convex game is not empty. It is well-known that an arbitrary game v can be represented, using the unanimity games, in a canonical way as a difference of two totally positive (and hence, convex) games: $v=v_+-v_-$.

Consider the difference \div of two compact convex sets A, B , defined as $A\div B:=\{x:x+A\subset B\}$. The difference $A\div B$ may be empty, but if not, this set, is convex and compact. For any game v the core $C(v)$ (it may be empty) possesses the following representation (Danilov/Coshevoi, 2000): $C(v)=C(v_+)\div C(v_-)$. Moreover, for every representation $v=v_1-v_2$ of a game v as a difference of two convex games $C(v)=C(v_1)\div C(v_2)$.

Definition 1. Let A be a convex compact set. The **Steiner point** of A is defined by:

$$St(A) = \frac{1}{\sigma_n} \int_{S^{n-1}} up(u, A) d\omega,$$

where u is a variable unit vector, $p(u, A)$ is the value of the support function of A in direction u , $d\omega$ is an element of surface area of the unit sphere S^{n-1} , and σ_n is the volume of the n -dimensional unit ball.

The key properties of St are: $St(A) \in A$, and linearity, i. e.

$$St(\lambda_1 A_1 + \lambda_2 A_2) = \lambda_1 St(A_1) + \lambda_2 St(A_2).$$

Definition 2. Let v be an arbitrary TU game, and $v = v_1 - v_2$ be a representation of v as a difference of two convex games. The **solution** S is defined by formula: $S(v) = St(C(v_1)) - St(C(v_2))$.

The definition is correct, i. e. it does not depend on representation of v , and S possesses the following properties.

- (1) **Additivity.** For any two TU games u and v , $S(u+v) = S(u) + S(v)$.
- (2) **Efficiency.** For any game $v: \sum_{i=1}^n s_i(v) = v(N)$.
- (3) **Null player property.** Let i be a null player in a game v , i. e. $v(S \cup i) = v(S)$ for every coalition $S \subset N$. Then $S_i(v) = 0$.
- (4) **Anonymity.** Let v be a TU game, and π be a permutation of the set $N = \{1, 2, \dots, n\}$. Let the game πv be defined as: $\pi v(S) = v(\pi S)$. Then $S(\pi v) = \pi^* S(v)$, where π^* is the orthogonal transformation of R^n induced by π .

Theorem 1. The solution S coincides with the Shapley value: for every TU game v , $Sh(v) = St(C(v_1)) - St(C(v_2))$, where $v = v_1 - v_2$ is an arbitrary decomposition of v as a difference of two convex games. If v is convex, then $Sh(v) = St(C(v))$.

If a convex set A is d -dimensional polytope in R^n , then the Steiner point possesses a simple representation (Shephard, 1966). Let S^{n-1} denote the unit $(n-1)$ -sphere centered on the origin. For any vertex w of A denote by $Z(w, A)$ the subset of S^{n-1} consisting of all those unit vectors u which are normal to the support hyperplane $H(u, A)$ of A for which $H(u, A) \cap A = w$.

The external angle $\psi(w, A)$ of A at w is the ratio of $(n-1)$ -content of $Z(w, A)$ to the $(n-1)$ -content of S^{n-1} . Let $w_j, j=1, \dots, r$ be the vertices of A . Then $St(A) = \sum_{j=1}^r w_j \psi(w_j, A)$.

Corollary 1. Let v be a convex game. The Shapley value is the weighted sum of the extreme points of the core of the game, the weights being the external angles of the core at the corresponding extreme points.

For a given permutation π of the player set N the corresponding **marginal contribution vector** $m^\pi(v)$ is defined by $m^\pi(v)_{\pi(k)} := v(S_k^\pi) - v(S_{k-1}^\pi)$ for each player $k \in N$, where $S_0^\pi := \emptyset$ and $S_k^\pi := \{\pi(j) : j \leq k\}$ for each $k \in \{1, 2, \dots, n\}$.

It is well-known, that for a convex game the extreme points of its core are precisely the marginal vectors. This means, in particular, that the Shapley value of a convex game is the weighted sum of the extreme points of the core with the weights equal to $k(x)/n!$, where $k(x)$ is the number of marginal vectors defining precisely the extreme point x .

Theorem 2. For any convex game v

$$\psi(x, C(v)) = \frac{k(x)}{n!}.$$

Corollary 2. Let v be a convex game. Then

$$Sh(v) = \sum_{x \in \text{ex} C(v)} \frac{k(x)}{n!} x = \sum_{x \in \text{ex} C(v)} \psi(x, C(v)) x = St(C(v)).$$

Moreover, $\psi(x, C(v)) = \frac{k(x)}{n!}$.

Consider one more difference of convex compacts (see Demyanov and Rubinov, 1995). We restrict the definition to the convex polytopes.

Denote by $\text{ex} A$ the set of vertices of A , and by $N^O(a, A)$ the interior of the normal cone $N(a, A)$ of A at a . For the convex polytopes A, B the difference ATB is defined by:

$$ATB = \text{co}\{a - b : a \in \text{ex} A, b \in \text{ex} B, N^O(a, A) \cap N^O(b, B) \neq \emptyset\}.$$

Lemma 1. Let $v=v_1-v_2$ be a decomposition of an arbitrary game v as a difference of two convex games. Then $C(v_1)TC(v_2)=W(v)$, where $W(v)$ is the Weber set. This decomposition does not depend on representation of v .

Proposition 1.

$$C(v_1) \div C(v_2) \subset C(v_1)TC(v_2).$$

Corollary 3. (Weber, 1988). For every game v ,

$$C(v) \subset W(v).$$



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Multistage and Repeated Network Games

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Keywords: *Non-cooperative game, Equilibrium, Network formation, Dynamics.*

An n -person finite multistage network game is considered. Players are supposed to coordinate their actions with each other (form mutual agreements). Coordination of players is defined in terms of networks, in which players are identified as nodes, and mutual agreements represent links. Network formation mechanism is proposed.

Let $N = \{1, \dots, n\}$ be a finite set of players, and g be a network consisting from n nodes, and connecting players from the set N . By g^N we denote a set of all possible networks.

An n -person stage game is denoted by $G = \langle N, \{N_i\}_{i \in N}, \{G_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$, and a set of all such games is denoted by Γ . Players $i \in N$ form a network in this stage game, and a set N_i consists of players to whom i may offer a link. Let $g_i = (g_{i1}, \dots, g_{in})$ be a strategy of $i \in N$. Here

$$g_{ij} = \begin{cases} 1, & \text{if } i \text{ wants to form a mutual link with } j \in N_i \setminus i \\ 0, & \text{otherwise or if } j = i \end{cases}$$

A set of all strategies of i in stage game G is denoted by G_i . Suppose that link (i, j) is formed by mutual agreement of both players i and j (when $g_{ij} = g_{ji} = 1$).

In stage game G players $i \in N$ simultaneously choose their strategies $g_i \in G_i$, then a network g is formed, and players payoffs in stage game G are defined as $u_i : g^N \rightarrow R$, $i \in N$, where

$$u_i(g) = \sum_{j \in N \setminus i: (i,j) \in g} f_i(i, j).$$

Here $f_i(i, j)$ represents utility of player i from link $(i, j) \in g$. After that the game process moves to the next stage game in accordance with an a priori given single-valued mapping $\Phi: g^N \rightarrow \Gamma$, i. e. game G moves to a game $\Phi(g) \in \Gamma$, where g is the network which is realized in stage game G . After a finite number of stages, the game process stops. In a particular case, when for each $G \in \Gamma$, $\Phi(g) = G$ for all $g \in g^N$, we have a repeated network game.

A sequence of realized stage games $\{G\} \in \Gamma$ defines the game trajectory.

Players payoffs along this trajectory are defined as $\sum_{G \in \{G\}} u_i(g)$, $i \in N$, where g is the network which is realized in stage game $G \in \{G\}$.

A method of finding an optimal and time-consistent solution in the multistage network game is proposed. Results are illustrated by example.

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Sufficient Nonemptiness Conditions for Core of TU-Game

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Keywords: *Cooperative TU-game, Core, Balancedness, Sufficient conditions.*

Using well known linear program introduced in order to study the core of TU-game and optimality criterion we obtain a list of sufficient conditions under which the core is nonempty.

Consider TU-game (N, ν) where $N = \{1, \dots, n\}$, $\nu: 2^N \rightarrow R$, $\nu(\emptyset) = 0$. Let $\Omega = 2^N \setminus \{N, \emptyset\}$, $\lambda \in R^{2^n - 2}$, $e = (1, \dots, 1)^T \in R^n$ and $A = (A^S)_{S \in \Omega}$ is the $n \times (2^n - 2)$ incidence matrix of Ω . It is known that a game (N, ν) has a non-empty core $C(\nu)$ iff $\max\{\nu\lambda: A\lambda = e, \lambda \geq 0\} \leq \nu(N)$. For any basis B of A the linear program can be converted into the form

$$\max\{\nu_B B^{-1}e - (\nu_B B^{-1}D - \nu_D)\lambda_D: \lambda_B = B^{-1}e - B^{-1}D\lambda_D, \lambda_B, \lambda_D \geq 0\} \quad (1)$$

Assume that $B^{-1}e \geq 0$ and the following conditions

$$\nu_B B^{-1}e \leq \nu(N), \quad \nu_B B^{-1}D \geq \nu_D \quad (2)$$

hold. Then $\bar{\lambda} = (\bar{\lambda}_B, \bar{\lambda}_D)$ with $\bar{\lambda}_B = B^{-1}e$, $\bar{\lambda}_D = 0$ is an optimal solution of problem (1) and

$\nu\bar{\lambda} \leq \nu(N)$. Moreover, the payoff vectors x^i , $i \in N$, where

$$x_j^i = \begin{cases} x_j^*, & i \neq j \\ \nu(N) - \sum_{k \in N \setminus i} x_k^*, & i = j \end{cases}$$

$x^* = B^{-1}\nu_B$, are the core allocations. Therefore, for any arbitrary selected feasible basis we can describe the corresponding subset of cone of balanced games. Some of such

subsets are the extensions of known classes of TU-games. The conditions (2) can be also obtained by means of dual linear program. But the above way allows to formulate (2) in terms of balanced sets and use their properties for analysis of such conditions. The following theorem provides the conditions (2) for $B=(A^{N \setminus i})_{i \in N}$. If ν satisfies the conditions

$$\begin{aligned} \sum_{i \in N} \nu(N \setminus i) &\leq (n-1)\nu(N), \\ \nu(S) &\leq \frac{(|S|+1-n) \sum_{i \in S} \nu(N \setminus i) + |S| \sum_{i \in N \setminus S} \nu(N \setminus i)}{(n-1)}, \quad S \in \Omega, |S| \leq n-2, \end{aligned} \quad (3)$$

then $C(\nu) \neq \emptyset$ and x^* is determined by

$$x_i^* = \frac{\sum_{j \in N \setminus i} \nu(N \setminus j) - (n-2)\nu(N \setminus i)}{(n-1)}, \quad i \in N.$$

The inequality (3) together with the concavity conditions

$$\nu(S) - \nu(S \setminus i) \geq \nu(N) - \nu(N \setminus i), \quad S \in \Omega, i \in S,$$

implies that $C(\nu) \neq \emptyset$.

Assume now that $A \ni A^N$ and $B \ni A^N$. Then the first condition in (2) holds as equality and $x^* \in C(\nu)$. Next theorem provides the second condition in (2) for such basis. Let $B = ((A^{\{i\}})_{i \in H}, (A^{N \setminus i})_{i \in N \setminus \{H \cup r\}}, A^N)$, where $H \in 2^N \setminus \{N\}$, $r \in N \setminus H$. Then the second condition in (2) is equivalent to

$$\nu(S) \leq |S \setminus H| \nu(N) + \sum_{i \in S \cap H} \nu(i) - \sum_{i \in S \setminus H} \nu(N \setminus i), \quad r \notin S, \quad (4)$$

$$\nu(S) \leq (|H|+2-n)\nu(N) - \sum_{i \in H \setminus S} \nu(i) + \sum_{i \in N \setminus \{H \cup S\}} \nu(N \setminus i), \quad r \in S, \quad (5)$$

and x^* is determined by

$$x_i^* = \begin{cases} \nu(i), & i \in H \\ \nu(N) - \nu(N \setminus i), & i \in N \setminus \{H \cup r\} \\ (|H|+2-n)\nu(N) + \sum_{j \in N \setminus \{H \cup r\}} \nu(N \setminus j) - \sum_{j \in H \setminus S} \nu(j), & i = r \end{cases}$$

The system (4)-(5) characterizes such class of TU-games that at least one extreme point of imputation set $I(\nu)$ or dual imputation set $I^*(\nu)$ or core cover $I(\nu) \cap I^*(\nu)$ belongs to core.

Note that the set of games satisfying (4)-(5) contains T -simplex games, dual simplex games ([1]), zero-normalized monotonic games with veto player, clan games ([4]) (in particular, big boss games).

Let (π_1, \dots, π_n) be the ordering of N . For basis containing the columns A^N , $A^{\{\pi_1\}}$, $A^{\{\pi_1, \pi_2\}}$, ..., $A^{\{\pi_1, \pi_2, \dots, \pi_k\}}$, where $2 \leq k \leq n-1$, the conditions (2) define TU-games with some convexity behaviour. For $k=n-1$ we obtain such class of games that at least one extreme point of Weber set belongs to core. This class contains the weak permutationally convex games ([5]) (in particular, permutationally convex games ([2]) and games satisfy the CoMa-property ([3])). For basis corresponding to graph described in ([5]) we obtain the set of games containing the generalized permutationally convex games but does not coincides with him.

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Discrete-Time Bioresource Management Problem with Asymmetric Players

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Keywords: *Bioresource management problem, Fish wars, Nash bargaining solution*

Model with two players

Let two players (countries or fishing firms) exploit the fish stock during infinite time horizon. The dynamics of the fishery is described by the equation

$$x_{t+1} = (sx_t - u_{1t} - u_{2t})^\alpha, x_0 = x, \quad (3)$$

where $x_t \geq 0$ -- the size of the population at a time t , $s \in (0,1)$ -- natural death rate, $\alpha \in (0,1)$ -- natural birth rate, $u_{it} \geq 0$ -- the catch of player i , $i = 1,2$.

We suppose that the utility function of country i is logarithmic and players differ in their discount factors. Then the players' net revenues over infinite time horizon are:

$$J_i = \sum_{t=0}^{\infty} \delta_i^t \ln(u_{it}), \quad (4)$$

where $0 < \delta_i < 1$ -- the discount factor for player i , $i = 1,2$.

The main question arising here is how to construct the value function for the cooperative solution in the case when players have different discount factors. In [1] it was supposed that the cooperative value function is the sum of the individual payoff functions of cooperative players. Here we construct the schemes how to determine the joint discount factor in order to construct the cooperative payoff.

Let's denote $V_i(x, \delta_i)$ -- Nash equilibrium payoff of the player i and $V(x, \delta)$ -- joint cooperative payoff with joint discount parameter δ .

First, we show that the joint discount factor for the case when cooperative payoff is distributed proportionally among the players exists. We find the conditions on δ to satisfy the inequalities

$$\frac{\delta_i}{\delta_1 + \delta_2} V(x, \delta) \geq V_i(x, \delta_i), i = 1,2.$$

Second, we suppose that the cooperative payoff is distributed in the portion $\gamma V(x, \delta)$ and $(1 - \gamma)V(x, \delta)$, where γ is a parameter. We find the conditions on δ and γ to satisfy the inequalities

$$\gamma V(x, \delta) \geq V_1(x, \delta_1), (1 - \gamma)V(x, \delta) \geq V_2(x, \delta_2).$$

As a result we have the set of admissible parameters δ and γ . To construct the solution we propose to use the Nash bargaining scheme, i.e. to solve the next optimization problem

$$(\gamma V(x, \delta) - V_1(x, \delta_1))((1 - \gamma)V(x, \delta) - V_2(x, \delta_2)) \rightarrow \max_{\delta, \gamma}.$$

The conditions for analytical solution of this problem are constructed. In other cases the solution can be found numerically.

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General Models of Multi-Criteria Optimization with Quality Criteria

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Keywords: *Quality criteria, Decision making problem, A space of preferences, Pareto-dominance, Conclusive family of coalition*

We study mathematical models of multi-criteria optimization with quality criteria. By a quality criterion we mean a function from a set of alternatives into some linearly ordered set which is a scale for given criterion. The main problem is a description of the space of preferences, associated with certain group decisions.

A general model of multi-criteria decision making can be presented in the form of a system

$$\langle A, (q_j)_{j \in J} \rangle \quad (1)$$

where A is a set of all alternatives and $q_j, j \in J$, are criteria for valuation of these alternatives (so-called local criteria). A criterion q_j is called a quality one if its scale is some linearly ordered set $\langle C_j, \sigma_j \rangle$, i.e. a chain.

Further we consider some rules for preferences or group decisions. A well known rule for preferences is Pareto-dominance \leq^{Pd} which leads to the notion of Pareto-optimal alternative. However the set of all Pareto-optimal alternatives can be very large hence there is an important problem of a contraction of it. We will study various kinds of decision rules which lead to contraction the set of Pareto optimal alternatives. Appropriate methods are based on some additional information concerning of local criteria.

1. Ordered product of the family of scales. Assume we have an information concerning of relative importance of local criteria in the form of their partial ordering \prec . Then we can construct the ordered product of the family of chain $(\langle C_j, \sigma_j \rangle)_{j \in J}$, where

the set J is ordered by \prec . It is known [1] in the case the ordered set $\langle J, \prec \rangle$ satisfies descending chain condition then we obtain an order relation ω on A which contains the Pareto-dominance \leq^{Pd} . Hence, in this case, the set of maximal elements under order ω is involved in the set of Pareto-optimal alternatives. Consider now two special kinds of this construction.

A) The relation \prec is empty (that is an information concerning of relative importance of local criteria is absent). Then ω coincides with Pareto-dominance \leq^{Pd} .

B) The relation \prec is a linear order. Then $\langle A, \omega \rangle$ is a linearly ordered set and ω becomes the lexicographical order.

Remark that variants A and B are extreme and occur very seldom in real situations of decision making with many criteria.

2. A family of conclusive coalition. Another method for constructing of preference relation is based on the notion of conclusive coalition of criteria. Assume we have additional information which indicates important groups of criteria in the following sense. Subset S of criteria is called a conclusive coalition if for any two alternatives $a, b \in A$ the preference between them is defined by preferences under all criteria of S . Let W be a family of all conclusive coalition. Then we can define a preference relation ρ on the set of alternatives A by the formula:

$$a \leq^\rho b \Leftrightarrow \{j \in J : q_j(a) \leq q_j(b)\} \in W \quad (2)$$

We consider the following conditions for a family of conclusive coalition W (see [2]):

C1. Axiom of non-void: $W \neq \emptyset$;

C2. Axiom of stability: $S \in W, T \supseteq S \Rightarrow T \in W$;

C3. Axiom of anti-complementary: $S \in W \Rightarrow S' \notin W$.

An axiom characteristic for group decision (2) satisfying to conditions (C1)-(C3) is found.

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Game-Theoretic Approach to Insider Trading Modeling: One-Stage Bidding with non-zero Bid-Ask Spread

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Keywords: *Insider trading, Bid-Ask spread, Incomplete information*

This work is concerned with game-theoretic modeling of stock markets with asymmetric information. In the pioneer work of B. De Meyer and H. Moussa Saley [1] the model of bidding with asymmetric information on the side of one stockmarket agent was introduced. This market model is based on the theory of repeated games with incomplete information. In this paper arbitrary bids are permitted.

It appears more natural to allow only integer bids proportional to a minimal currency unit. This kind of models has been investigated by V. Domansky and V. Kreps since 2007 (for instance [3],[4]). Solving discrete models generates a lot of combinatorial difficulties. The only solved models are the model of unlimited duration and some special cases. So for games modeling n -stage ($n < \infty$) bidding the complete solution was obtained only for one-stage bidding games (see [5]).

In all these models each player proposes bid=ask.

In 2007 B. De Meyer [2] introduced a general trading mechanism to demonstrate that informational asymmetries are driving the prices. One of realizations of this mechanism is an assumption of non-zero bid-ask spread.

This work is a first appearance of model with non-zero bid-ask spread. We treat a generalization of one-stage model studied in [5].

Here we consider a one-stage model of double sealed-bid auction between two stockmarket agents where one unit of risky asset (a share) is traded. The random liquidation price of a share is determined by a chance move before bidding and may take two values: the positive integer m with the probability p and 0 with the probability $1-p$. Informational asymmetries of the market manifests in the fact that institutional

investors are better informed than private investors. Player 1 is a risk neutral informed investor, he receives a message about the liquidation share price. Player 2 is not informed about random selection of the share price, he just knows p . Insider also knows p . Everybody knows who is informed.

Both players propose simultaneously their bids, i.e. they post bid and ask for a share. Transaction occurs from seller to buyer by bid price. The aim of each player is to maximize the liquidation value of his final portfolio.

The bid-ask spread s is fixed by rules of auction. The model is reduced to the zero-sum game $G_1^{m,s}(p)$ with lack of information on the side of Player 2.

The analysis of one-stage model is really instructive: it demonstrates combinatorial difficulties of discrete model and allows us to construct a bridge to solving multi-stage bid-ask models.

We expand the recurrent approach to computing game value and optimal strategy of Player 2 introduced in [5] to the case of non-zero bid-ask spread. We analyze the complicated structure of optimal strategies of both players. The results are illustrated by means of computer simulation.

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Repeated Incomplete Information Games with Countable State Space: Broken \sqrt{N} -Law

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Keywords: *Variation of bounded martingales, Repeated games with incomplete information, Information theory, Entropy*

We consider repeated two-person zero-sum games with incomplete information where Player 1 is informed about the state $k \in K$ and Player 2 is not. We denote such game by $\Gamma_N(\rho)$, where N is the number of repetitions and ρ is the prior probability distribution over the state space K . At the end of the game Player 1 (maximizer) receives from Player 2 the non-averaged sum of per step gains.

These games were introduced by R. Aumann and M. Maschler in the sixties. Since their pioneer works the asymptotic behavior of game values $V_N(\rho)$ as $N \rightarrow \infty$ is one of central problems of the field. R. Aumann and M. Maschler proved (see [1]) the following theorem.

Theorem: *If the state space K and the action spaces are finite sets, then*

$$v_N(\rho) = N \cdot \text{Cav } v^{\text{NR}}(\rho) + \delta_N(\rho),$$

where $\text{Cav } v^{\text{NR}}(\rho)$ is the concavification of the value of the one-stage non-revealing game where both players know only the distribution ρ ; the error term $\delta_N(\rho)$ is nonnegative and

$$\delta_N(\rho) \leq C\sqrt{N}.$$

Note that for a wide class of games $\text{Cav } v^{\text{NR}}(\rho) = 0$ (for stock market games with this property see [4] and [5]) and therefore δ_N becomes the leading term.

A lot of papers was devoted to δ_N studies. This problem is closely connected with the analysis of posterior probabilities variations. Denote by $\rho_n(k)$ the posterior probability of the state $k \in K$ induced by informed player actions up to the stage n

($\rho = \rho_0$). The sequence $\{\rho_n\}$ represents the evolution of Player 2 beliefs about the actual state during the game. The N -term variation of posterior probabilities is defined in the following way:

$$V_N(\{\rho_n\}) = E \sum_{n=0}^{N-1} \sum_{k \in K} |\rho_{n+1}(k) - \rho_n(k)|, \quad V_N(\{\rho_n\}) = E \sum_{n=0}^{N-1} \sum_{k \in K} |\rho_{n+1}(k) - \rho_n(k)|$$

To estimate δ_N from above R. Aumann and M. Maschler proved that

$$\delta_N(\rho) \leq C_1 V_N(\{\rho_n\}) / N,$$

where the sequence $\{\rho_n\}_{n=0}^N$ is induced by the optimal strategy of Player 1. Further they estimated the variation

$$V_N(\{\rho_n\}) \leq \sqrt{N} \Lambda(\rho), \quad \text{where} \quad \Lambda(\rho) = \sum_{k \in K} \sqrt{\rho(k)(1 - \rho(k))} \quad (*)$$

(this inequality holds for an arbitrary informed player strategy).

Thus the fastest growth of δ_N is connected with the maximal possible variation of posterior probabilities. For deeper connections between a variation and repeated incomplete information games see [7] and [3].

The above mentioned reasons inspired the investigation of posterior probabilities variation by itself. The main problem in this direction is to determine the maximal possible variation growth as $N \rightarrow \infty$ for a given prior distribution ρ . In the case of a two-element state space a comprehensive asymptotic analysis of the maximal variation problem was carried out in the papers of J.-F. Mertens & S. Zamir [6] and B. De Meyer [2].

Our aim is to consider the opposite extreme case of the countable state space K . The main questions we are going to answer are the following:

- What does it happen when the sum in (*) diverges? Does the divergence imply an anomalous variation growth or the \sqrt{N} -bound always holds?
- Can the anomalous behavior of the variation lead to the anomalous growth of the error term δ_N of the corresponding game?

We introduce the following measure of prior distribution uncertainty

$$Z_\epsilon(\rho) = \sum_{k \in K} \rho(k) \left(\ln \left(\frac{1}{\rho(k)} \right) \right)^{\frac{1}{2} - \epsilon}$$

Note that $Z_{-1/2}$ coincides with Shannon's entropy. It occurs that the finiteness of Z_0 defines the precise set of priors for which \sqrt{N} -law holds. More formally:

Theorem: Let ρ be the prior distribution over K

• if $Z_0(\rho) < \infty$, then \sqrt{N} -law holds, moreover

$$V_N(\{\rho_n\}) \leq \sqrt{2N}(Z_0(\rho) + \sqrt{\pi/4}).$$

• if for some $\varepsilon \in (0, 1/2)$ we have $Z_\varepsilon(\rho) = \infty$, then \sqrt{N} -law is broken, i.e.,

there exists the sequence of posterior probabilities $\{\rho_n\}_{n=0}^\infty$ with prior ρ such that for all $\varepsilon' < \varepsilon$

$$\limsup_{N \rightarrow \infty} \frac{V_N(\{\rho_n\})}{N^{\frac{1}{2} + \varepsilon'}} = \infty.$$

To answer the second question we construct the game $\mathcal{G}_N(\rho)$. It is N -stage zero-sum repeated game with incomplete information. Players have infinite action spaces but one-stage pay-offs are bounded. We prove the anomalous behavior of the error term in $\mathcal{G}_N(\rho)$:

Theorem: If $Z_\varepsilon(\rho) = \infty$ for some $\varepsilon \in (0, 1/2)$, then

$$\forall \varepsilon' < \varepsilon \quad \limsup_{N \rightarrow \infty} \frac{\delta_N(\rho)}{N^{\frac{1}{2} + \varepsilon'}} = \infty.$$

Our approach is based on the combination of the ideas of B. De Meyer's work [2] and tools of C. Shannon's information theory. We note that information theory tools seem to be very useful not only for the considered problem but for different problems of incomplete information games. The reason is the following: these tools are very natural for the problems related with posterior probabilities and thus for the incomplete information games too, where the posterior probabilities are the objects of the most importance. The construction of \mathcal{G}_N is inspired by the results of J. Mertens and S. Zamir [7] on normal games.

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Formation of the Coalitional Structure of Games: Farsighted Coalitional Stability and Its Modifications

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Keywords: *Stability set, Uncovered set, Top cycle, Consistent set*

For game in which the set of alternatives is finite and nonempty and there are odd number of voters (players), each with linear, transitive, and asymmetric strong preferences three solution concepts are always nonempty: the top cycle, the uncovered set, and the stability set at least. In this paper we consider the uncovered set for subspace.

On Fair Income Allocation in Acyclic Network Systems

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Keywords: *Network games, Income redistribution, Cooperation acyclic graphs.*

Optimal performance of multi-agent systems is a pivotal problem of the modern economics. Performance quality of such systems can be described by a set of numerical characteristics reflecting the way these systems operate. These characteristics could include profit, cost, gross margin, quantity or number of produced commodities, and will be referred to henceforth as Objective Functions (OF). Without losing the generality, the maximization formulation will be considered.

There are two possible approaches to selecting the optimal strategy. The first one (henceforth ‘collective’ approach) is used when an overall system performance is considered and individual agents’ performances are not the main focus of the research. In this case, the most common tool employed for the system optimization is the non-linear or linear programming, usually integer or mixed. The second approach implies optimization considered from each of the agents’ point of view (henceforth referred to as ‘individual’). With this approach, the system optimization tool is the game theoretic methods. The first approach (overall optimization) is equivalent to the second approach (game) when all agents form one large coalition. The value of overall maximized OF is not less than the sums of the individual OFs obtained by individual players or coalitions as solutions of a game. It is tempting to suggest that agents form one coalition and then divide the total payoff among them. However, some players (or coalitions of players)

might be better off pursuing strategies different from the strategy formulated for the overall optimization for ‘one large coalition’ game.

This work continues the research reported last year on the GTM2011 conference by Gurvich and Schreider (2011). A wide range of real-life situations in resource management (which include some optimal water, gas and electricity allocation problems) can be modeled by the network LP optimization problems in which the major constraints are related to the delivery capacity.

As it was shown by Boros et al. (1997), for the systems whose network structure is represented by an acyclic graph a stable optimal solution exists. Moreover, this result is extended to the systems whose coalitional structure is modeled by the Berge normal hypergraphs.

Optimality is treated here as a LP solution for the coalition of all players populating this acycling graph, while stability means that the total optimal income of the system can be distributed among players in such a way that no individual player, or coalition of players, can be better off playing differently.

This approach is illustrated by two real-life examples:

- (i) the agricultural system in the Murray-Goulburn catchment where the water is delivered by the gravity channels between set of producers and
- (ii) the natural gas allocation system in Southeastern Australia.

The problems of non-uniqueness of such stable income allocations and selecting the "optimal" one are also discussed.

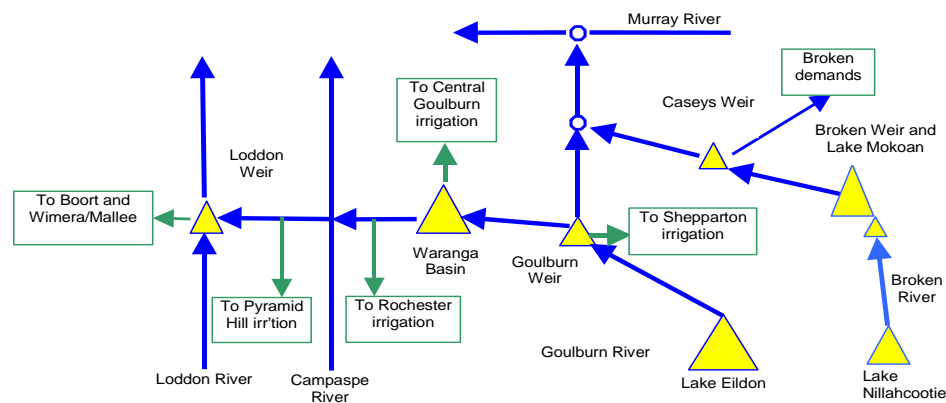


Figure 1. Simplified network representation of the Goulburn –Murray water allocation system

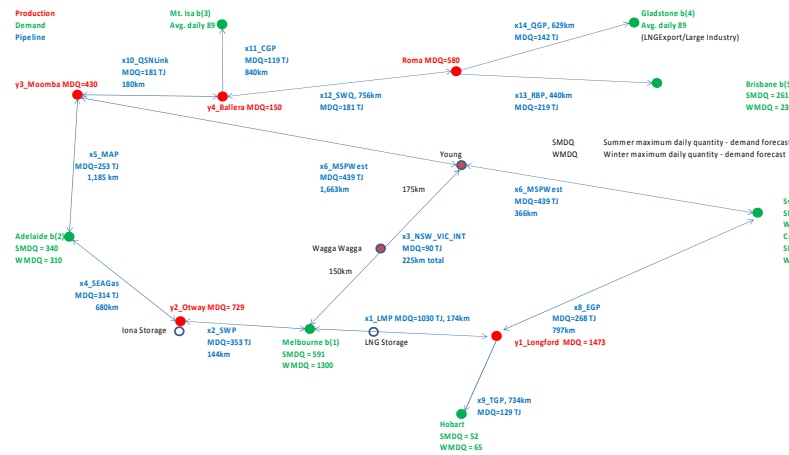


Figure 2. Natural gas distribution system in Southeastern Australia

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Evaluation of Anagement Participating In Mergers Performance

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Keywords: *Agency problem, Property redistribution, Management efficiency, Mergers, Management opportunistic behavior, MBO*

Company's market position depends on the quality of decisions made by its management. The classical and neoclassical economic theory is based on the fact that managers act only in the interests of shareholders' maximizing their wealth.

The market for corporate control is an important tool for monitoring activities of top managers because it shows the objective results of their activities.

Using game theory methods, we can model the mechanism of property redistribution in process of corporate conflict between owners and managers of the company, and subsequently we can compare the simulation and actual transactions results involving the companies operating in the oil services market of Perm region in 2004-2008.

Consider the redistribution of property model in which shareholders and managers of the target company which is preparing to merge try to reallocate their assets. Both of them own company's assets, the disposal of managers is also options to buy the most profitable assets. We assume that all players maximize their welfare, information is complete and imperfect, players have no incentive to collude.

Target company may sell their assets to managers of the company and can buy the assets of the target company managers. After the transaction award the target company's managers may exercise the option and buy out the most profitable assets of the company at a discount. If managers do not exercise the option, these assets are transferred to the owners.

Once managers and owners of the target company agree on the distribution of assets among themselves, they may still remain shareholders of their business, or they may sell their assets to another company's managers and shareholders who act as independent players. Thus, at the same time two auctions are held, one of which is the auction where target company's managers' assets are sold, another one is the auction where target company's shareholders' assets are sold. There are two buyers in both cases: managers and owners of the company-buyer. The seller chooses the best offer and sells all of its assets to this buyer. If customers call the same price, the seller may sell half of its assets to managers, and the other half to the owners of the target company. After the bargaining asset owners will reach the profit in proportion to their share in business.

If the acquirer pays a premium for all assets of target company, merger would be profitable for owners and managers of the target company, while shareholders and managers of the company-buyer suffer losses.

If the acquirer is willing to pay premium only for those assets, which provide a strategic advantage, and the remaining assets are traded at a discount, then target company managers do not agree to sell their assets and retain their positions and control. Outsiders will get significant merger premium, however, their gain will be smaller than in the case when all the assets of the corporation were sold. If the assets of outsiders will move completely under the control of buyer's managers, the latter will redistribute a significant part of welfare in their favor. This option is very typical for mergers and acquisitions in the oil industry. Managers of Russian oil companies tend to concentrate in the hands of a big block of shares that entitle them to manage cash flows.

It should be noted that the game outcome depends mainly on the information asymmetry. Less informed player would have smaller number of possible strategies and it would make the game less favorable for him.

Model presented in the article coincides with the real mechanism of CHURS and SGD acquisition by the LUKOIL company. In fact, bargaining between owners and managers of target companies, initiated by the owners gave no results, because in one case, managers have refused to sell their assets (SGD), and in another case, the price requested by the owners of the company was too high, and CHURS managers failed to complete the MBO. Within the proposed model there is no opportunity to explain the fact that managers refuse to exercise the option. This decision is the result of the model imperfections: when the target company's managers decide whether to exercise the

option or not, they take into account only the assets' purchasing price, and do not expect that in future, these assets can provide strategic advantages.

However, in reality SGD managers resold the most profitable (oil) assets of the company to LUKOIL managers. As the result both management teams have received lucrative contracts, merger premium, salary increase and high positions.

Simulation of the negotiations process between owners and managers of the target company can be concluded that none of them can not concentrate control over all assets of the corporation prior to start of negotiations with the acquirer, but the most profitable assets of the company are coming under the control of its owners.

The model constructed confirmed that in fact shareholders do not have possibility to block unprofitable deals and risk of losing high positions and salaries is more important for top managers in evaluating the efficiency of merger than shareholders' welfare concern (even if managers themselves own the company's assets). It rises the question of the effectiveness of managerial participation in business capital motivation policy. Overall, this suggests that the evaluation of Russian transactions is more correct within the concept of corporate stakeholders, and not within the traditional concept of maximizing shareholders' welfare.

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Delegation in Long-Term Relationships

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Keywords: *Delegation, Signalling, Reputation.*

A central question in the design of organisations, which has been much discussed in the literature over the last decades, is how to allocate decision rights to subordinate agents (e.g. Holmström, 1977, 1984, Aghion and Tirole, 1997, Alonso and Matouschek, 2008). The general problem is that while agents often may have better information about the profitability of certain projects – or at least be able to obtain such information – this does not necessarily imply that they will always opt for the projects which are most preferred by the principal. A possible reason for this, which we will focus on in the sequel, is that agents may be biased and therefore disagree with the principal on what project to choose in a given state of the world.

An immediate direct effect of such biases, if unknown to the principal, is that the expected profit from delegation decreases due to too frequent choices of suboptimal projects (for a discussion of such instances see, for example, Jensen and Meckling, 1992). A more indirect but related effect, which arises in repeated interactions, derives from reputation concerns of the agents. In particular, agents may have an incentive to be perceived as unbiased and therefore strategically distort their information; in such cases, eliciting information from the agent via communication (Crawford and Sobel, 1982) may be problematic (see, for example, Morris, 2001, or Sobel, 1985).

In the present paper, we combine both of the above mentioned aspects and analyse the consequences of delegation by an uninformed principal in a repeated interaction. In particular, we introduce asymmetric information about the agent's preferences while the agent's decisions are also subject to reputational concerns.

For the purposes of our argument, we assume that there are two types of agents: a) agents who are unbiased or loyal and b) agents who are biased with regard to the principal's preferences. Both the principal's and the agent's preferences depend on the state of the world, which is determined by a chance move that can only be observed by the agent. However, while the loyal agent agrees with the principal in his project choice, the biased agent always prefers to implement a different project which he privately benefits from. In the one-shot interaction where the principal can either delegate or take an uninformed decision himself, the optimal delegation decision therefore depends on the principal's prior belief about the type of the agent and the principal's relative payoff of taking an uninformed decision.

In a repeated interaction, however, things change as the agent's project choice in the first interaction may also signal information about his type. Accordingly, the principal's delegation decision now is not only driven by the agent's informational advantage but also by an incentive to learn about the agent's type. In particular, if the principal delegates to the agent in the first period, this results in a signalling game where the agent first chooses a project (thereby potentially revealing his type) and the principal then decides whether or not to delegate again in the second period. Thus, being *ex post* informed about the congruence of the agent's decision with his own preferences, the principal can condition his delegation decision in period two on this congruence.

The foreseen reaction of the principal in period two, in turn, induces additional reputation concerns for the agent. Regarding these incentives, we assume that second period wages are determined endogenously depending on the agent's reputation, i.e. the principal's belief about the agent's type (his bias) at the beginning of period two conditional on the agent's project choice in period one. In particular, we assume that the agent's wage is an increasing function of his reputation.

Although the assumption about endogenous wages slightly complicates the analysis, the resulting overall two-period game is amenable to a common backward induction argument. Relying on standard equilibrium selection arguments (Cho and Kreps, 1987), we characterise the equilibria of the resulting signalling game and report the results of comparative statics based on changes in the prior probability that the agent is biased and the agent's relative loss from choosing the disliked project if he is biased. The analysis shows that the principal delegates more often in the first period of the repeated interaction than in the one-shot game. At first sight, this may seem surprising as the principal pays for the agent's gain in reputation (through an increased wage in period

two) which will render delegation less attractive in comparison to standard models of reputation. However, the cost of inefficient delegation in the first period and higher wages in period two is off-set by the expected benefit from being able to discriminate between types in combination with the induced reputation incentives for the biased agent to align his period one choice with the principal's preference.

More specifically, in the analysis we find multiple equilibria for the signalling game depending on the prior belief about the agent's bias and the specific relation between wages and beliefs. In particular, if wages (given beliefs) are low compared to the loss of the biased agent from implementing a disliked project in period one, we obtain a separating equilibrium, where the agent's type is perfectly revealed after the first period. In this case, the principal expands the agent's initial discretion in comparison to a one-shot interaction in order to be able to sort between types which at least allows him to avoid the undesired decision of the biased agent in period two. By contrast, if wages are high, both types choose the project which is preferred by the principal in period one (pooling equilibrium). In this case, the principal strictly prefers delegation to centralisation in order to exploit the agent's informational advantage. In intermediate cases, we get mixed equilibria in the signalling game, with the biased agent (and the principal) randomising between the two possible options, and higher levels of delegation than in the one-period case.

Interestingly, the different equilibria imply different effects on aggregate welfare compared to the repeated one-period interaction. In particular, (a) the pooling equilibrium, which effectively mirrors the repeated interaction, is neutral with respect to welfare, (b) welfare is decreased in the mixed equilibrium, and (c) the separating equilibrium may lead to either an increase in welfare or a decrease, with the direction of the effect depending on whether the biased agent's benefit from centralisation in period two – compared to an opportunistic choice in period one – is larger (increase) than the principal's benefit from the agent's opportunistic choice in period one — compared to centralisation in period two – or not (decrease).

What is more, while wages have no direct impact on aggregate welfare, as they affect only the redistribution of payoffs, they still matter, namely through their (indirect) influence on equilibria. In particular, transferring all benefits to the agent (as is the case if the market for agents is perfectly competitive) reduces both the range of undesirable separating and mixed equilibria. Thus, in the present setting, giving all bargaining power to the agents maximises aggregate welfare.

Finally, we want to emphasise that, from an applied point of view, the results suggest that the delegation of authority to agents, while potentially inefficient from a one-shot perspective, offers a reasonable way to separate the wheat from the chaff in long term relations (or to at least push the biased agents to align their project choices with the principal's preferences). In fact, an empirical analysis of data from the German Socio-Economic Panel (SOEP) shows that, in accordance with our theoretical results, workers in a temporary status experience less autonomy in their decisions than workers in a regular employment relationship.

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The Game-Theoretical Model of the Choice of Service with Big Losses

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Keywords: *Game-theoretical model, Service model, Non-linear big losses.*

We will consider a company that offers its services to clients in order fulfillment, and several service options for different conditions. The first method of service is characterized by a fairly large queue waiting for service, but a fixed fee for the order. The second method is characterized by a lack of service queue, i.e. customer immediately gets a service without having to wait, but must pay a fixed fee plus a fee for each unit of time of the order. It should be noted the feature of incoming customers. If the delay of order is over the certain time then each client has great loss.

Denote by η_1 and η_2 times which clients spend on the service by selecting the first or second mode, respectively. Each of these times is the sum of two terms: a waiting service time η_{11}, η_{21} and maintenance time η_{12}, η_{22} . The values of η_1, η_2 are random variables with density functions with exponential distribution with parameters $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$ respectively and μ_2 many times more than μ_1 .

The client goes to a company in order to receive service and its natural tendency is to minimize operating costs, which consist of several components: a fixed fee for the order, pay per unit time of the order, as well as great losses incurred by the customer at a delay of the order.

Let denote by ν_1 and ν_2 the average costs of care in selecting the first or second mode, respectively. We introduce an indicator

$$I\{t, T\} = \begin{cases} 1, & \text{if } t \leq T, \\ 0, & \text{if } t > T, \end{cases}$$

that makes the problem non-linear. Also, we denote by l_1 and l_2 the costs of care in one of service devices of the company.

For the second mode of maintenance costs are divided into two terms:

$l_2 = l_{21} + l_{22} \eta_{22}$, where l_{21} is a fixed fee, l_{22} is a cost per unit time of the order.

In addition to standard maintenance costs for clients there exists a risk of incurring large losses when the delay of order exceeds a certain limit set by each client.

The process of selecting the optimal behavior under choosing a service can be formulated in terms of game theory.

Define the non-antagonistic game in normal form:

$\Gamma = \langle N, \{p_i\}_{i \in N}, \{K_i\}_{i \in N} \rangle$, where

$N = \{1, \dots, n\}$ - set of players,

$\{p_i\}_{i \in N}$ - set of strategies, $p_i \in [0, 1]$,

$\{K_i\}_{i \in N}$ - set of payoff functions.

$$K_i = -(p_i V_{1i} + (1 - p_i) V_{2i}) = -(p_i (V_{1i} - V_{2i}) + V_{2i}),$$

where p_i is the probability of player i choose the first mode, $1 - p_i$ - is the probability of player i choose the second mode respectively.

Define customer specific loss of waiting service r .

$V_{1i} = r(E\eta_i^{(11)} + E\eta_i^{(12)}) + l_1 + R_i EI\{\eta_i^1, T_i\}$ - player i expected loss for the first mode service,

$V_{2i} = (r + l_{22})E\eta_i^{(22)} + l_{21} + R_i EI\{\eta_i^2, T_i\}$ - player i expected loss for the second mode service, where r is a cost associated with missed opportunities per unit time that is spent on maintenance,

R_i is loss incurred by the client i if the delay of the order more than T_i , $i = 1, \dots, n$.

This means that to a certain point of time the customer can wait its order and the delay of the order will not affect on its wealth, but from some point of the time the customer has the big loss from the delay.

In this paper we formulate and prove the theorem for finding optimal strategies for players behavior when choosing a service provided with non-linearity of the loss function. The Nash equilibrium is found.

Time-Consistency Problem in Transportation Games

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Keywords: *Network, transportation games, network flows, time-consistency problem.*

The m -person transportation game over the network $G=(X,N)$ is considered. The players $i \in \{1, \dots, m\} = M$ have to reach the target vertexes $z_i \in X$, from given initial vertexes $y_i \in X$, with minimal costs. The corresponding cooperative TU game is investigated, as solution the Shaply Value is choosen. Suppose $x^* = (y=x_1^*, \dots, z=x_l^*)$ is cooperative trajectory minimizing total costs. Computation of Shaply Value for subgames along the cooperative trajectory x^* shows time-inconsistency of Shapley Value.

The regularisation procedure leading to the time-consistent Shaply Value is proposed.

Location-Price Game in the Market of two Products

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Keywords: *Credibility Location-price game, Subgame perfect Nash equilibrium, Hotelling's duopoly model*

The study of firms behavior in the location-price competition was started by H. Hotelling [1]. He considered competition in a linear market with uniform customer distribution. Since then, a large number of researches have been made on Hotelling's model and its different modifications.

We study a case of a linear market with two product types. Let assume, that a product of the 1st type is sold by a firm located in α . Each customer is going to buy products of two types and competitive behavior of two firms selling the 2nd type product is examined.

The model is formulated as a two-stage game. In the first stage locations x_i are chosen simultaneously and then prices p_i are set on the second stage of the game. The solution concept is defined as subgame perfect Nash equilibrium. Customers are distributed over the given segment $[0,1]$ with known density function $f(x)$. Some examples for the given $f(x)$ are presented.

We also examine a case when firms can't be considered as perfectly reliable, i.e. a firm can refuse to serve a customer with the probability p . Firm failure may occur due to a wide variety of reasons, e.g. product shortage or firm's time closing. The impact of possible serve failure on optimal location is examined

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Guaranteed Strategies for Alternative Pursuit Games

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Keywords: *Alternative pursuit, Sliding motion, Finite number of updates.*

A game is called alternative if it may be terminated by the pursuer on any of two given terminal manifolds, where the payoffs of Boltza type differ only in their terminal parts [1,2]. Assuming that the optimal strategies and trajectories for both corresponding games with given alternatives known, we discuss how to construct pursuit strategies that provide guaranteed outcomes in the original game.

Let $z_P(t) \in \mathbb{R}^{n_P}$ and $z_E(t) \in \mathbb{R}^{n_E}$ obey the equations

$$\begin{aligned}\dot{z}_P(t) &= f_P(z_P(t), u_P(t)), \quad z_P(0) = z_P^0, \\ \dot{z}_E(t) &= f_E(z_E(t), u_E(t)), \quad z_E(0) = z_E^0,\end{aligned}$$

where $t \geq 0$, $u_P(t) \in U_P \subset \mathbb{R}^{m_P}$, $u_E(t) \in U_E \subset \mathbb{R}^{m_E}$, U_P and U_E are compact sets, $f_P: \mathbb{R}^{n_P} \times U_P \rightarrow \mathbb{R}^{n_P}$ and $f_E: \mathbb{R}^{n_E} \times U_E \rightarrow \mathbb{R}^{n_E}$, z_P^0 and z_E^0 are initial positions of the players. Let $z = (z_P, z_E) \in Z = \mathbb{R}^N$, $N = n_P + n_E$, and

$$\dot{z}(t) = f(z(t), u_P(t), u_E(t)), \quad z(0) = z^0,$$

where $z^0 = (z_P^0, z_E^0)$, $f(z, u_P, u_E) = (f_P(z_E, u_P), f_E(z_E, u_E))$. We assume that f is jointly continuous and locally Lipschitz with respect to z , and satisfies the extendability condition; see, e.g., [3].

In G_l , $l \in L = \{a, b\}$, for a given continuous trajectory $z(\cdot) = \{z(\tau)\}_{\tau \geq 0}$, $z^0 \in Z_l^i$ and sufficiently small $\varepsilon > 0$, the outcome functional is equal to

$$\begin{aligned}
P_l^\varepsilon(z(\cdot)) &= \begin{cases} \tau_l^\varepsilon + K_l(z(\tau_l^\varepsilon)), & \text{if } \tau_l^\varepsilon = \tau_l^\varepsilon(z(\square)) < \infty \\ \infty & \text{otherwise} \end{cases} \\
\tau_l^\varepsilon(z(\square)) &= \begin{cases} \min\{\tau : z(\tau) \in Z_l^{f,\varepsilon}\} & \text{if } \exists \tau : z(\tau) \in Z_l^{f,\varepsilon} \\ \infty & \text{otherwise} \end{cases}
\end{aligned} \tag{1}$$

where $Z_l^{f,\varepsilon}$ is an exterior ε -neighborhood of the terminal manifold Z_l^f , $K_l \in C_Z^2$. In G , $z^0 \in Z^i = Z_a^i \cap Z_b^i$, the actual terminal alternative l depends on $z(\cdot)$ and the payoff is defined as $P_l^\varepsilon(z(\cdot))$ if $\tau_l^\varepsilon(z(\cdot)) = \min(\tau_a^\varepsilon(z(\cdot)), \tau_b^\varepsilon(z(\cdot))) < \infty$ (see (1)).

For a given initial state $z^0 \in Z_l^i$, partition $\Delta = \{t_0, t_1, \dots\}$ of \mathbb{R}^+ and pursuit strategy $U_P \div u_P$, let $Z_P(z^0, U_P, \Delta)$ be a set of stepwise solutions of the inclusion

$$\dot{z}(t) \in \text{co}\{f(z(t_i), u_P(t_i), u_E) : u_E \in U_E\}, \tag{2}$$

where $t \in [t_i, t_{i+1}]$, $i \in \mathbb{N}$, $t_0 = 0$, $t_i \rightarrow i \rightarrow \infty$. $Z_P(z^0, U_P, \Delta)$ includes continuous functions $z(\cdot) : \mathbb{R}^+ \rightarrow Z$, that have absolutely continuous restrictions to $[0, \theta]$ for any $\theta > 0$ and meet (2) for almost all $t \in [0, \theta]$.

The guaranteed outcome in G_l for P is evaluated as [3]

$$\tilde{P}_l(z^0) = \lim_{\varepsilon \rightarrow 0+} P_l^\varepsilon(z^0), \tag{3}$$

where $\tilde{P}_l^\varepsilon(z^0) = \min_{U_P} P_l^\varepsilon(z^0, U_P)$,

$$\begin{aligned}
\tilde{P}_l^\varepsilon(z^0, U_P) &= \lim_{|\Delta| \rightarrow +0} \tilde{P}_l^\varepsilon(z^0, U_P, \Delta), \quad |\Delta| = \sup_{i \in \mathbb{N}} (t_{i+1} - t_i), \\
\tilde{P}_l^\varepsilon(z^0, U_P, \Delta) &= \sup_{z(\cdot) \in Z_P(z^0, U_P, \Delta)} P_l^\varepsilon(z(\cdot)).
\end{aligned}$$

Let $\hat{P}_l(z^0)$ be a similar index for E , and V^l be the value function that represents a guaranteed outcome for both agents, $V^l(z^0) = \tilde{P}_l(z^0) = \hat{P}_l(z^0)$, $\forall z^0 \in Z_l$. Denote the optimal feedback strategies, trajectories and durations in G_l as $U_P^l \div u^l(z) : Z \rightarrow U_P$, $U_E^l \div u^l(z) : Z \rightarrow U_P$, $z_P^l(\cdot, z^0)$, $z_E^l(\cdot, z^0)$ and $\tau^l(z^0)$. Let

$$V^{ab}(z^0) = \min(V^a(z^0), V^b(z^0)), \quad z^0 \in Z^i.$$

We call a pursuit strategy with a memory $U_{P \div u_P}(t, \{z(\tau)\}_{\tau \in [0, t)}): R^+ \times C[0, \infty)$ guaranteed in G if $\lim_{\varepsilon \rightarrow 0+} \tilde{P}^\varepsilon(z^0, U_P) \leq V^{a|b}(z^0)$. Obviously, if $l(z^0) \in L$ meets the condition $V^{l(z^0)}(z^0) = V^{a|b}(z^0)$, the strategy $U_{P \div u_P}^{l(z^0)}(z)$ is guaranteed. However, if P updates the targeted terminal alternative at every state with the strategy $U_{P \div u_P}^{l(z)}(z)$, $V^{l(z)}(z) = V^{a|b}(z)$, a sliding motion on a focal line [4] may lead to a worse outcome.

Our talk will discuss several pursuit strategies with a finite number of updates for the targeted terminal alternative and their guaranteed features.

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On the Conditions for the Stability of Cooperative Agreement in Linear-Quadratic Differential Games

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Keywords: *Differential games, Linear-quadratic differential games, Time-consistency problem, Irrational behaviour proofness condition*

A particularly important aspect of the theory of cooperative differential games is the dynamic stability of cooperative agreements. In the following, we distinguish two different cases: the individually rational and the irrational behaviour of players. The former one is expressed through the time consistency property of a cooperative solution [1, 2, 3, 4]. This means that the employed imputation distribution scheme is such that the cooperative players' payoffs always dominate their non-cooperative counterparts. The latter is called the irrational behaviour proofness condition [5] and refers to the situation where a player behaves irrationally, for example by cancelling the cooperative agreement. In this case it is desired that the worst case players' payoffs are not worse than those corresponding to the non-cooperative case.

In this contribution we consider a class of linear-quadratic differential games and study different cooperative agreement stability conditions. In particular, we discuss different imputation distribution schemes and formulate structural conditions on the differential game under study which guarantee the existence of a time-consistent distribution procedure. These results are compared with the equal gain splitting rule proposed by I. Curiel [6].

One important question is whether it is possible to formulate a cooperative linear-quadratic differential game such that the cooperative agreement is stable with

respect to both the individually rational and the irrational behaviour of players. Some initial results on this topic are discussed and a simple example is considered.

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Journals in Game Theory

DYNAMIC GAMES AND APPLICATIONS

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Revenue Management by a Patient Seller

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Keywords: *Mechanism design, Dynamic auctions, Revenue management.*

We consider a classic setting as in Myerson (1981): a seller faces N risk-neutral buyers. However, the seller is assumed to be more patient than the buyers. We restrict attention to selling mechanisms where the price can change over time, and buyers can buy at any point in time. A static auction with an optimally chosen reserve at time 0 is included as a special case, but, as we show, in general is not optimal. The optimal solution involves delays, an auction at the initial time as well as the auction at the final time, and sales at posted prices at intermediate times.

We study dynamic pricing games (DPG), where the seller commits to a price schedule, beginning with a high price, and then reducing it until the item is sold. We consider a perishable good: the time of sale is exogenously constrained by the lifespan of the good. The price schedule is the object of seller's design, with the purpose of maximizing the discounted expected revenue.

Buyers draw their valuations from the same distribution, and we restrict attention to symmetric equilibria. Buyers are forward-looking. At any point in time, a buyer can either purchase at the standing price, or wait. If several buyers are willing to purchase at the same time, the seller conducts an auction among those buyers. We assume an open auction, or, equivalently, a Vickrey auction. But all our results generalize to other revenue equivalent formats, including first-price sealed-bid (or Dutch).

A variety of plausible pricing behaviors are allowed, as we put essentially no restrictions on the form of the price schedule beyond monotonicity. For example, we allow the price to drop instantaneously at any point in time. We also allow the price to remain constant over an interval, so that the good is effectively removed from the market over that period. A classic, instantaneous auction at time 0 with a reserve price is also included as a special case.

Our main result can be summarized as follows. If the seller is less (or equally) patient, the instantaneous auction at time 0 with a reserve price is optimal, as in Myerson (1981). However, if the seller is more patient than the buyers, then he or she can do better by committing to a reserve price that declines over time. Thus, an optimal solution involves delays. We use optimal control to characterize the solution. As we show, the seller can equivalently implement the price schedule via a time of sale function, prescribing which buyer types should buy at different points in time. Any point of strict monotonicity of this function corresponds to sale at a posted price, while any flat segment corresponds to an auction.

To gain further insights into the structure of the solution, we first consider a relaxed problem, with the monotonicity constraint removed. We show that the optimal solution will involve auctions at the initial time 0 and the final time T . Otherwise, sales will occur at posted prices. The posted prices are shown to be always above the static optimal reserve price, but at the final time, the price is discontinuously reduced below it. Moreover, the optimal solution is continuous; implying that temporarily removing the good from the market is not optimal. A numerically computed example suggests that our optimal dynamic mechanism outperforms the instantaneous optimal auction by about 11%.

We next characterize the solution under the monotonicity constraint. We show that almost all properties of the relaxed-optimal solution carry over. The only possible exception is that now we cannot rule out auctions at intermediate times, precisely when the monotonicity constraint is binding.

On the technical side, our general analysis is based on the Lebesgue rectification theorem, a new technique that we believe can be applied in other contexts. We parameterize the solution as a curve in the plane, with potential discontinuities filled out by connecting line segments. The aforementioned theorem implies that, for monotone functions, the coordinates in such a parameterization can be chosen as absolutely continuous functions. This enables us to impose the monotonicity constraint in a standard fashion, in terms of the derivatives of the coordinate functions, and invoke the Maximum Principle to characterize the solution.

The solution is then shown to be continuous. At all points where it is strictly monotonic, we show that the Hamiltonian satisfies the same first-order condition as in the unconstrained case. As the properties of the solution chiefly rely on this first-order condition, we can transfer the results from the unconstrained case to the constrained one with little changes.

Choosing Products in Social Networks

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Keywords: *Social networks games, Join the crowd property, Nash equilibria, Finite improvement property*

Social networks are a thriving interdisciplinary research area with links to sociology, economics, epidemiology, computer science, and mathematics. A flurry of numerous articles and recent books, see, e.g., [2], testifies to the relevance of this field.

In [1] we introduced a new threshold model of a social network in which nodes (agents) influenced by their neighbours can adopt one out of several products. We use here this model to understand and predict the behaviour of the consumers (agents) who form a social network and are confronted with several alternatives (products). To this end we associate with each such social network a natural strategic game.

Preliminaries

Assume a set $\{1, \dots, n\}$ of players, where $n > 1$. A strategic game for n players, written as $(S_1, \dots, S_n, p_1, \dots, p_n)$ consists of

- a non-empty set S_i of strategies ,
- a payoff function $p_i : S_1 \times \dots \times S_n \rightarrow \mathbb{R}$ $p_i : S_1 \times \dots \times S_n \rightarrow \mathbb{R}$,

for each player i

Fix a strategic game $G := (S_1, \dots, S_n, p_1, \dots, p_n)$. We denote $S_1 \times \dots \times S_n$ by S , call each element $s \in S$ joint strategy, denote the i th element of s by s_i , and abbreviate the sequence $(s_j)_{j \neq i}$ to s_{-i} . Occasionally we write (s_i, s_{-i}) instead of s . Finally, we abbreviate $\times_{j \neq i} S_j$ to S_{-i}

We call a strategy s_i of player i a best response to a joint strategy s_{-i} of his opponents if

$$\forall s'_i \in S_i \quad p_i(s_i, s_{-i}) \geq p_i(s'_i, s_{-i})$$

Next, we call a joint strategy s a Nash equilibrium if each s_i is a best response to s_{-i} , that is, if

$$\forall i \in \{1, \dots, n\} \quad \forall s'_i \in S_i \quad p_i(s_i, s_{-i}) \geq p_i(s'_i, s_{-i})$$

In turn, we call a strategy s_i of player i a better response given a joint Strategy s if $p_i(s'_i, s_{-i}) > p_i(s_i, s_{-i})$

Following [4] a path in S is a sequence (s^1, s^2, \dots) of joint strategies such that for every $k > 1$ there is a player i such that $s^k = (s'_i, s_{-i}^{k-1})$ for some $s'_i \neq s_{-i}^{k-1}$. A path is called an improvement path if it is maximal and for all $k > 1$, $p_i(s^k) > p_i(s^{k-1})$ where i is the player who deviated from s^{k-1} .

The last condition simply means that each deviating player selects a better response. A game has the finite improvement property (FIP) if every improvement path is finite. Obviously, if a game has the FIP, then it has a Nash equilibrium—it is the last element of each path.

Finally, a game is called weakly acyclic (see [3]) if for every joint strategy there exists a finite improvement path that starts at it.

Social networks

We are interested in strategic games defined over social networks. We use the model recently introduced in [1].

Let $V = \{1, \dots, n\}$ be a finite set of agents and $G = (V, E, w)$ a weighted directed graph with $w_{ij} \in [0, 1]$ being the weight of the edge (i, j) . Given a node i of G we denote by $N(i)$ the set of nodes from which there is an incoming edge to i . We call each $j \in N(i)$ a neighbour of i in G . We assume that for each node i such that $N(i) \neq \emptyset$, $\sum_{j \in N(i)} w_{ji} \leq 1$. An agent $i \in V$ is said to be a source node in G if $N(i) = \emptyset$.

Let \bar{P} be a finite set of alternatives or products. By a social network we mean a tuple $S = (G, \bar{P}, P, \theta)$, where P assigns to each agent i a non-empty set of products $P(i)$ from which it can make a choice. For $i \in V$ and $t \in P(i)$ the threshold function θ yields a value $\theta(i, t) \in (0, 1]$. The threshold $\theta(i, t)$ should be viewed as agent's i resistance level to adopt a product t .

Given a social network S we denote by $source(S)$ the set of source nodes in the underlying graph G . Fix a social network $S = (G, \bar{P}, P, \theta)$. Each agent can adopt a product from his product set or choose not to adopt any product. We denote the latter choice by t_0 .

Social network games

With each social network S we associate a strategic game $G()$ among agents defined as follows, where c_0 is some given in advance positive constant:

- the set of strategies for player i is $S_i := P(i) \cup \{t_0\}$,
- For $i \in V$, $t \in P(i)$ and a joint strategy s , let

$$N_i^t(s) := \{j \in N(i) \mid s_j = t\},$$

i.e., $N_i^t()$ is the set of neighbours of i who adopted in s the product t .

The payoff function is defined as follows.

- for $i \in source(S)$,

$$p_i(s) = \begin{cases} 0 & \text{if } s_i = t_0 \\ c & \text{if } s_i \in P(i) \end{cases}$$

- for $i \notin source(S)$,

$$p_i(s) = \begin{cases} 0 & \text{if } s_i = t_0 \\ \sum_{j \in N_i^t(s)} w_{ji} - \theta(i, t) & \text{if } s_i = t, \text{ for some } t \in P(i) \end{cases}$$

We call such games social network games. These games satisfy the following join the crowd property:

Each payoff function p_i depends only on the strategy chosen by player i and the set of players who also chose his strategy. Moreover, the dependence on this set is monotonic.

Main results

First we introduce a modification of the notion of FIP using an auxiliary concept. By a scheduler we mean a function f that given a joint strategy s that is not a Nash equilibrium selects a player who does not play in s a best response. An

improvement path $\rho = (s^1, s^2, \dots)$ conforms to a scheduler f if for all $k \geq 1, s^{k+1} = (s'_i, s^k_{-i})$, where $f(s^k) = i$. We say that a strategic game has the uniform FIP if there exists a scheduler f such that all improvement paths ρ which conform to f are finite. Below we call a Nash equilibrium s non-trivial if for some $i, s \neq t_0$ and trivial otherwise.

Here is a summary of our main results

Theorem 1.

- There exists a social network S for which $G(S)$ has no Nash equilibrium.
- Suppose the underlying graph of S is a DAG. Then $G(S)$ has a non-trivial Nash equilibrium and has the FIP.
- Every two players social network game has the FIP.
- Suppose the underlying graph of S is a simple cycle with ≥ 3 nodes Then $G(S)$ does not need to have the FIP. However, it does have the uniform FIP.
- Suppose the underlying graph of $S = (G, \bar{P}, P, \theta)$ has no source nodes. nodes Then $G(S)$ does not need to have the FIP.

For a product $t \in \bar{P}$, let $X_t := \bigcap_{m \in \mathbb{N}} X_t^m$, where

- $X_t^0 := \{i \in V \mid t \in P(i)\}$,
- $X_t^{m+1} := \{i \in V \mid \sum_{j \in N(i) \cap X_t^m} w_{ji} \geq \theta(i, t)\}$.

Then $G(S)$ has a non-trivial Nash equilibrium iff there exists a product t such that $X_t \neq \emptyset$. (A trivial Nash equilibrium always exists.)

- There exists a social network game that is weakly acyclic but does not have the uniform FIP.
- There exists a social network game that has a Nash equilibrium but is not weakly acyclic.

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A Competitive Prediction Game

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Keywords: *Game of timing, Optimal stopping, Silent duels, Infinite game.*

In [1] models of two person prediction games were formulated. The problem is closely related to the silent duels and the auction bidding. When the player's pay-off function in the game of timing depends on the moment of the player's action than for some class of function the equilibrium exists in randomize strategies. The computational difficulties were discussed by Trybula and Radzik in [3] in the chapter VI. The aim of this research is to investigate a price of successful prediction on the value and equilibrium in the game. The problem is relevant to the game on sale which was analyse by Teraoka [2].

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Owen-Type Value for Games with two-Level Communication Structures

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Keywords: *TU game, Coalition structure, Communication graph, Owen value, Myerson value*

In this paper we introduce a new singleton solution concept for the class of games with two-level communication structures. A game with a two-level communication structure is TU game endowed with both coalition and communication structures when a communication structure is a two-level communication structure that relates fundamentally to the given coalition structure. It is assumed that cooperation (via bilateral agreements between participants) is only possible either among the entire a priori unions of a coalition structure or among single players within a priori unions. No communication and therefore no cooperation is allowed between single players from distinct elements of the coalition structure. The introduced value modifies the Owen value for games with coalition structures to the class of games with two-level communication structures. As in Owen (1977) the payoffs of the players are determined by applying the Shapley value twice. First, the Shapley value is applied to the Myerson restricted game (with respect to the communication structure between unions) of Owen's quotient game between the unions. This gives the total payoff to the players of each union. Next to obtain the individual payoffs, within each union the Shapley value is applied to a game on the players within the union. To construct the game within a union, first a game is obtained by applying Owen's procedure to find such a game, but taking into account the communication structure between the unions. Next we take a restriction

of this game with respect to the communication structure within the union. The Shapley value is applied to this restriction.

The new Owen-type value for the class of games with two-level communication structures is characterized by four axioms, two on the level of the communication structure between the a priori unions, and two on the level of the communication structures within the a priori unions. We also show that the Owen value and the Aumann-Dr ze value for games with coalition structures, the Myerson value for games with communication structures and the equal surplus division solution appear as special cases of this new value for particular two-level communication structures.



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INTERNATIONAL GAME THEORY REVIEW

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Polar Representation of the Shapley Value for Nonatomic Games

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Keywords: *Nonatomic cooperative game, Generalized Owen extension, Multiplicative Aumann-Shapley expansion, Polar form, Shapley value, Polar representation formula*

Introduction

The paper deals with the polar representation formula for the Shapley value, established under rather general assumptions in [3]. In order to simplify the proof of this formula for some special classes of games we continue our investigation on a generalization of the Owen extension [2] for the regular polynomial games, started in [4]. Main attention is paid to the nonatomic cooperative games. Our approach is based on the principal result from [4], demonstrating that the above-mentioned generalized Owen extension coincides with the multiplicative Aumann-Shapley expansion for some types of nonatomic games, including those from pNA , which have bounded polynomial variation. This coincidence makes it possible to calculate the Aumann-Shapley expansion in a straightforward manner by applying corresponding generalized Owen extension. To complete the new proof of the polar representation formula for the Shapley value of nonatomic homogeneous game we exploit the famous generalized Owen integral formula from [1], given in terms of the multiplicative Aumann-Shapley expansion.

Generalized Owen extension of regular polynomial game

Let (Q, d) be an arbitrary nonempty metric compactum with distance function d . Denote by B its Borel σ -algebra and consider a collection V of set functions $v: B \rightarrow \mathbf{R}$ satisfying the requirement $v(\emptyset) = 0$. As usual, a triplet $\Gamma = (Q, B, v)$ with $v \in V$ is said to be a cooperative game (with elements of Q being players, and elements

of B treated as their coalitions). Below, we consider, mostly, the case of infinite cooperative games (when Q is an infinite set).

To remind the definition of regular polynomial games under consideration, we recall first some notations and definitions from [3,4]). Fix $e \in B$ and denote by $H(e)$ a set of finite B -measurable partitions of e . Put $H = \bigcup_{e \in B} H(e)$. For any $\eta = \{e_i\}_{i \in \Omega} \in H$ with $|\Omega| = m$, and $v \in V$ denote by $v(\eta) = v(\{e_i\}_{i \in \Omega})$ a polynomial m -difference, defined by the formula

$$v(\eta) := \sum_{\omega \subseteq \Omega} (-1)^{|\Omega| - |\omega|} v(\bigcup_{i \in \omega} e_i), \quad (1)$$

where, as usual, the term $|\omega|$ denotes the number of elements of a finite set ω .

A game $v \in V$ is said to be *totally positive* if $v(\eta) \geq 0$ for any $\eta \in H$. Convex cone of the totally positive games is denoted by V_+ . Further, put $V := V_+ - V_+$ and introduce the norm of polynomial variation $\|v\|_0 := \inf \{u(Q) + w(Q) \mid u - w = v, u, w \in V_+\}$, $v \in V$. It is known [3], that V , endowed with the partial order $u \geq_0 v \Leftrightarrow u - v \in V_+$ along with the norm $P \cdot P_0$, is norm complete and Dedekind complete vector lattice with the norm compatible with partial order \geq_0 : monotone order convergence implies monotone norm convergence.

Following standard notations of the vector lattice theory, for any function $v \in V$, denote by $v^+ = v \vee 0$, $v^- = -v \vee 0$, and $|v| = -v \vee v$ the positive, negative, and total variations of v , respectively (as usual, $u \vee w := \sup \{u, w\}$, and $u \wedge w := \inf \{u, w\}$ w.r.t. the partially ordered vector space (V, \geq_0)). Let F be a collection of all closed subsets of Q . The basic type of games we are going to deal with is given by the following definition.

Definition 1. 1. A game $v \in V$ is said to be *regular*, if its total variation $|v|$ meets the requirement: $|v|(\{e_i\}_1^m) = \sup \{|v|(\{f_i\}_1^m) \mid f_i \subseteq e_i, f_i \in F, i = 1, \dots, m\}$ for any partition $\eta = \{e_i\}_1^m \in H$. A set of regular games is denoted by rV .

Definition 2. A game $v \in rV$ is called a *regular polynomial game of order n* if all the polynomial $n+1$ -differences of v are equal to zero: $v(\{e_i\}_1^{n+1}) = 0$ for any $\{e_i\}_1^{n+1} \in H$. Denote by rV^n a space of regular polynomial games of order n and

introduce a space $rV^{(n)}$ of regular homogeneous polynomial games of order n , consisting of those games from rV^n that are disjoint with the space rV^{n-1} :
 $rV^{(n)} := \{v \in rV^n \mid |v| \wedge |u| = 0, u \in rV^{n-1}\}.$

Polar representation of the Shapley value: nonatomic games

For simplicity, in the sequel we consider the case $Q=[0,1]$. Put $rpV := \cup_{n=1}^{\infty} rV^n$ and denote by $pvNA$ those games from pNA that belong to rpV (roughly speaking, pNA is a vector space, spanned by the powers of nonatomic probabilistic measures; for more details see [1]). By applying axiomatic characterization of the generalized Owen extension P_v [4], and theorem H from [1], known as generalized Owen integral formula, we get the following result.

Theorem 1. For any $v \in pvNA$, and for any $e \in B$, directional derivative

$$\partial P_v(t, e) := \frac{d}{d\tau} P_v(t\chi_Q + \tau\chi_e), \text{ calculated at } \tau = 0, \text{ exists at each point } t \in [0,1].$$

Moreover, this derivative is integrable as a function of $t \in [0,1]$. In addition, for any $n \geq 1$, the Shapley value $\Phi : pvNA \cap rV^{(n)} \rightarrow rV^1$, and derivatives of P_v in the direction of χ_e satisfy the equalities

$$\Phi(v)(e) = \int_0^1 \partial P_v(t, e) dt, \quad e \in B.$$

Remind [3], that for any $v \in rV^{(n)}$, there exist a poliadditive and symmetric function $\psi_v : B^n \rightarrow \mathbf{R}$, whose diagonal restriction coincides with v :

$$\psi_v(e, \dots, e) = v(e), \quad e \in B.$$

This function ψ_v is called a polar form of the game v . In the paper, a new approach for to study the interconnection between the polar forms and Shapley values of nonatomic homogeneous games is proposed. In particular, a short proof of the polar representation formula for the Shapley value in case $v \in pvNA$, based on the Theorem 1 and straightforward calculation of the directional derivatives of generalized Owen extension P_v , is given.

Theorem 2. For any homogeneous game $v \in pvNA$ it holds

$$\Phi(v)(e) = \psi_v(e, Q, \dots, Q), \quad e \in B.$$

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DYNAMIC GAMES AND APPLICATIONS

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Models of Capacity and Electricity Market

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We consider stylized models of Russian electricity and capacity market: pay-as-bid and uniform price versions of capacity market design. We compare the equilibrium outcomes with the optimal capacity structure. The optimal structure is a solution of the social welfare optimization problem. We show that the market equilibrium corresponds to the optimal capacity structure under conditions of pure competition, full rationality, and completely informed agents in the market. However, under more realistic assumptions, selection of the optimal structure is unlikely. We provide the auction design that realizes such selection of capacity and does not require any additional information of each producer besides his own production costs.

Under centralized planning, determination of the optimal capacity structure for an electricity sector is an optimization problem. Its solution includes capacities that cover the demand with minimal total costs. Consider a formal model. The demand within one annual period is price inelastic and characterized by the maximum value \bar{M} and the duration of load curve $M(\tau)$ (DLC below). The inverse function $\tau(M)$ determines the share of the time within the period when the required capacity exceeds M . In practice, the curve $M(\tau)$ is calculated based on consumption in previous periods. Assume that capacity is produced by standard small generators (units, for instance each unit produces one kWt) and the DLC is a piece-wise constant function with the values corresponding to the integer numbers of such units. Let A be the set of units under

selection, each $i \in A$ is characterized by fixed and variable costs c_i^f and c_i^v . The fixed costs are determined as follows: $c_i^f = \frac{r \cdot OC_i}{(1 - e^{-rT_i})}$, where OC_i is the cost of construction for unit i , r is the discounting rate, and T_i is durability of the unit. The problem is to choose \overline{M} units $(a_1^*, \dots, a_{\overline{M}}^*)$ that cover the demand given by DLC $M(\tau)$, $M(0) = \overline{M}$, with minimal total costs. For a given collection $(a_1, \dots, a_{\overline{M}})$, the costs $C(a_1, \dots, a_{\overline{M}}) \stackrel{\text{def}}{=} \sum_{l=1}^{\overline{M}} (c_f^{a_l} + \tau(l)c_v^{a_l})$ where $\tau(l)$ is the time when unit a_l is employed within the period. This time is determined by the inverse function to $M(\tau)$. Thus, the formal problem is to find

$$(a_1^*, \dots, a_{\overline{M}}^*) = \arg \min_{(a_1, \dots, a_{\overline{M}}) \in A} C(a_1, \dots, a_{\overline{M}}) \quad (1)$$

An algorithm for the optimal choice is as follows. At the first stage, find a unit that minimizes the cost $(c_f^a + c_v^a)$ (since $\tau(1)=1$). Consider stage i when units $\overline{a}_1, \dots, \overline{a}_{i-1}$ already have been determined. Find

$$\arg \min_{a \in A} (c_f^a + \tau(i)c_v^a) \quad (2)$$

If there is a solution $\overline{a} \notin \{\overline{a}_1, \dots, \overline{a}_{i-1}\} \stackrel{\text{def}}{=} A_{i-1}$ then let $\overline{a}_i = \overline{a}$. Otherwise, consider DLC $\min(M(\tau), i)$ obtained from the original DLC $M(\tau)$ by limiting the maximal demand with i units. For every unit a that is optimal in the set $A \setminus A_{i-1}$ in the sense of problem (2) with some $j \leq i$ substituted for i , compute the minimum cost necessary to cover the load $\min(M(\tau), i)$ with the set $A_{i-1} \cup \{a\}$: order units in this set according to c_v^a (from lowest to highest) and find $\sum_{l=1}^i (c_f^{a_l} + c_v^{a_l} \tau(l))$ for reordered collection (a_1, \dots, a_i) . Denote this value by $C^i(A_{i-1}|a)$. Finally, choose $\overline{a}_i = \arg \min_{a \in A \setminus A_{i-1}} C^i(A_{i-1}|a)$, set $A_i = A_{i-1} \cup \{\overline{a}_i\}$ and pass to stage $i+1$.

Proposition 1. The set $\overline{A}_{\overline{M}}$ obtained according to the algorithm described above and arranged in order of increasing c_v^a is a solution of the problem (1).

Russian electricity and capacity market includes two main components: a day ahead market (DAM) and an auction for competitive selection of capacities (CSC). Since 2011, the rules of the Russian capacity market are similar to the uniform price auction. We show that the competitive equilibrium structure is the solution to the problem of the optimal capacity structure. One important disadvantage of this mechanism is that the optimal bid of agent a depends not only on his fixed cost but also on the value of his expected profit at the DAM that in its turn depends on parameters of other agents. Thus, the CSC would not necessarily select the agents with minimum fixed costs, but might select those who are better informed about parameters of competitors.

Now consider the auction design that provides selection of the optimal capacity structure $\overline{A_M}$ under mild assumptions on the structure of the market and information of participants. Each producer a can propose one unit with parameters c_f^a, c_v^a that are his private information. He submits an offer (p_f^a, p_v^a) . An auctioneer processes the offers according to the above given algorithm and selects \overline{M} units $(a_1, \dots, a_{\overline{M}})$. Each selected producer a_l bears the costs $c_f^{a_l} + \tau(l)c_v^{a_l}$ and is paid

$$p_f^{a_l} + \tau(l)p_v^{a_l} + \max_{i=1, \dots, \overline{M}} (p_f^{a_i} + \tau(i)p_v^{a_i} - p_f^{a_l} - \tau(i)p_v^{a_l})$$

This rule generalizes the uniform price auction design for a homogenous good to the capacity and electricity market. It provides selection of the optimal capacity structure under the following condition. For every $l=1, \dots, \overline{M}$, consider the market where unit a_l^* is removed from the set A . Let $(a_1, \dots, a_{\overline{M}})(l)$ be the optimal capacity structure for such market. Assume that

$$\max_i \left(c_f^{a_i(l)} + \tau(i)c_v^{a_i(l)} - c_f^{a_l^*} - c_v^{a_l^*} \tau(i) \right) = \max_i \left(c_f^{a_i^*} + \tau(i)c_v^{a_i^*} - c_f^{a_l^*} - c_v^{a_l^*} \tau(i) \right) \quad (3)$$

This relation means that the profit of unit a in the auction under honest revealing of the costs is equal to its maximal profit under manipulation of the costs in this unit's offer.

Proposition 2. Under condition (3), the Nash equilibrium strategy of each producer at such auction is to offer (c_f^a, c_v^a) . The selected collection $(a_1, \dots, a_{\overline{M}})$ is a solution of problem (1).

Thus, the given auction design provides selection of the optimal capacity structure with account of the fixed and variable costs of available capacities. Optimal behavior of each producer at such auction is very simple and corresponds to the revelation principle (Myerson, 1981). It does not require any additional information besides his own production costs. For implementation of this auction in practice it is necessary to specify the algorithm and the auction rules for a general form of a DLC and generators characterized by their lower and upper capacity values.



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Are Social and Entrepreneurial Attitudes Compatible? A Behavioral and Self-Perceptual Analysis

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Keywords: *Social entrepreneur, Experimental behavioral economics, Risk.*

Purpose – The aim of this paper is to analyze the compatibility between entrepreneurial and social attitudes. Specifically, we analyze if subjects with a more developed economic entrepreneurial attitude exhibit a less social attitude.

Design/methodology/approach – Our methodology integrates an economic experimental approach with a standard entrepreneurial intention questionnaire to analyze the interaction between entrepreneurial and social self-perceptions and behavior.

Findings – There is empirical evidence that experimental entrepreneurial behavior (characterized by detecting an opportunity and accepting risk to take an economic advantage from it in laboratory experiments) reduces the incentive for social behavior. However, this effect does not appear if just self-perceptions instead of experimental behaviors are considered.

Research limitations/implications – The social attitude of entrepreneurs may be overestimated in those empirical research studies based only on data obtained from entrepreneurs' answers to hypothetical questions in a survey.

Originality/value - To the best of our knowledge, this is the first paper presenting a laboratory experiment to represent the key features of entrepreneurial behavior instead of a case-control analysis to set differences in the experimental behavior of sub-samples of subjects defined in terms of their entrepreneurial motivation or experience.

Analysing the Folk Theorem for Linked Repeated Games

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Keywords: *Folk theorem, convex set, Minkowski sum, Pareto boundary.*

Results are obtained for the Minkowski sum of intersections and for the Pareto boundary of a Minkowski sum. These results are, via the Folk Theorem, of interest for the theory of the linking of repeated games.

Fix subsets U_1, \dots, U_m of \mathbb{R}^n . Let $H_k := \mathbb{R}_+^n \cap \text{Conv}(U_k)$;
 $H := \sum_{k=1}^m H_k$; $H_e := \mathbb{R}_+^n \cap \sum_{k=1}^m \text{Conv}(U_k)$.

It is easy to see that

$$H \subseteq H_e.$$

We study conditions under which $H \subset H_e$. Denoting by $\text{PB}(X)$ ($\text{PB}_w(X)$) the strong pareto boundary (weak pareto boundary) of a subset X of \mathbb{R}^n , let

$$\text{EXP} := \text{PB}(H) \setminus \text{PB}_w(H_e),$$

the set of *strong expansions points* of $\text{PB}(H)$. We study conditions under which

$$\text{EXP} = \emptyset \text{ (nowhere expansion);}$$

$$\emptyset \subset \text{EXP} \subset \text{PB}(H) \text{ (partial expansion);}$$

$$\emptyset \subset \text{EXP} = \text{PB}(H) \text{ (expansion everywhere).}$$

The obtained results are, via the Folk Theorem, of interest for the theory of the linking of repeated games.

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Open and Closed Loop Nash Equilibria in Dynamic Games with a Continuum of Players - Extended Abstract

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Keywords: *Dynamic games, Continuum of players, Open loop Nash equilibria, Closed loop Nash equilibria*

Nash equilibrium is the most important concept in noncooperative game theory.

When dynamic games are considered, researchers are usually examining two types of Nash equilibria - open loop Nash equilibria and closed loop Nash equilibria. These classes of equilibria are usually not equivalent in any sense, unlike closed and open loop solutions of deterministic optimal control problems, from which dynamic games originate.

In this paper an equivalence theorem between closed loop and open loop Nash equilibria is proven for a very large class of deterministic dynamic games with continuum of players and discrete time.

This result allows also to use Bellman equation to calculate open loop equilibrium as well as Pontryagin maximum principle to calculate closed loop profiles or to combine both methods. Each of these three procedures applied to games in which such an equivalence result does not hold would lead to erroneous results.

This paper generalizes author's papers concerning dynamic games with continuum of players: Wiszniewska-Matyszkiewicz [1], [2] and [3] in which only global state variables were considered and various equivalence results were proven between a dynamic Nash equilibrium profile and a sequence of equilibria in static games along this profile - decomposition theorems.

Such decomposition theorems can be generalized to obtain an equivalence theorem between open loop and closed loop Nash equilibria similar to the main result of

this paper (but restricted to substantially smaller class of games without private state variables).

Theoretical results are illustrated by examples of models of exploitation of an ecosystem by a large group of players.

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WORLD SCIENTIFIC



The Lexicographic Prekernel

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Keywords: *Cooperative game with transferable utilities, Prekernel, Least core, Lexicographic prekernel*

The lexicographic prekernel of a cooperative game with transferable utilities lexicographically minimizes the maximal surpluses of one player over another (rather than the excess vectors as would be the prenucleolus). This solution has not enough been studied: it is only known that it is non-empty efficient solution contained in both the least core and the prekernel, and may not contain the prenucleolus (Yarom, 1981).

In the presentation a combinatorial characterization of vectors belonging to the lexicographic prekernel that a rather weak analog of the known Kohlberg's (Kohlberg 1971) characterization of the prenucleolus with the help of balanced collections of coalitions. The difference is in the fact that, in contrary to the excess vector, the components of the maximal surplus vector are already maximal values of excesses separating pairs of players.

Nevertheless, this characterization of the lexicographic prekernel permits to prove its consistency. This fact is enough important, since till the present only unique enough sophisticated solution for TU games satisfying nonemptiness, efficiency, covariance, equal treatment property, consistency and not containing the prenucleolus was known (Orshan, Sudhoelter, 2003).

Subgame Consistent Cooperative Solutions in Stochastic Differential Games with Asynchronous Horizons and Uncertain Types of Players

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Keywords: *Cooperative stochastic differential games, subgame consistency, asynchronous horizons, payment distribution mechanism.*

This paper considers cooperative stochastic differential games in which players enter the game at different times and have diverse horizons. Moreover, the types of future players are not known with certainty. Subgame consistent cooperative solutions and analytically tractable payoff distribution mechanisms leading to the realization of these solutions are derived. This analysis widens the application of cooperative stochastic differential game theory to problems where the players' game horizons are asynchronous and the types of future players are uncertain. It represents the first time that subgame consistent solutions for cooperative stochastic differential games with asynchronous players' horizons and uncertain types of future players are formulated.

Stability in Cooperative Routing Games

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Keywords: *Vehicle routing problem, VRP, Dynamic cooperative game, Time consistency, Dynamic stability*

The theory of cooperative games allows to investigate possibilities of coordination of actions of the companies for the purpose of decrease in expenses for transportations. One of the most important object of research of the theory of cooperative games is characteristic function of game which reflects estimations of the guaranteed values of total expenses of the participants who formed a coalition. In many practical examples, application of exact methods for finding optimal routing plan for coalition of carriers is unacceptable due to big number of costumers. At the same time, application of heuristic methods generally doesn't allow to guarantee subadditivity of constructed characteristic function. By consideration of dynamic models of cooperation it is important to apply procedures of cost distribution procedures to provide stability of dynamic cooperative agreements.

Each company has set of customers and own resources, such as depot and park of vehicles. These companies consider various variants of association in a coalition for the purpose of decrease in expenses for transportations. Each coalition satisfies demand of customers for transport services of all companies-participants of a coalition, using incorporated resources. Thus, within the limits of cooperation in each coalition there can be a redistribution of customers between participants of this of coalitions, and, as an obvious consequence of it, change of routing plan of vehicles of each company in comparison with the routes planned without cooperation. In the conditions of cooperation of operative decision-making on routing of shared vehicles there is a problem of a lack of time for fast appointment of routes which would minimize total transport costs of a coalition as the given problem belongs to the class of NP complex.

It is possible to apply the well enough investigated approach to each coalition to the decision of a problem of routing of vehicles with several depots. After a finding of optimum routes for each possible coalition and calculation of the general transport costs there is a possibility to calculate value of characteristic function of cooperative game of routing. It is necessary to notice that use of heuristic algorithms for a finding of the minimum expenses of a coalition generally doesn't guarantee subadditivity of characteristic function. Therefore in problems of routing of vehicles application of special metaheuristic algorithms of construction optimum (or close to optimum) the routes providing subadditivity of characteristic function is necessary.

In given paper mathematical statement of a problem of cooperation of the carriers, the new approach to construction of characteristic function of dynamic cooperative game of routing of vehicles and algorithm of construction of the scheme of distribution of the expenses, providing stability of the cooperative agreement is offered.

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